Abstract—This paper presents optimal pricing design for demand response (DR) integration in the distribution network. In particular, we study the energy scheduling problem for a load serving entity (LSE) that serves two types of loads, namely inflexible and flexible loads. Inflexible loads are charged under a regular pricing tariff while flexible loads enjoy a dynamic pricing tariff that ensures cost saving for them. Moreover, flexible loads are assumed to be aggregated by several DR aggregators. The interaction between the LSE and its customers is formulated as a bilevel optimization problem where the LSE is the leader and DR aggregators are the followers. The optimal solution of this problem corresponds to the optimal pricing tariff for flexible loads. The key advantage of the proposed model is that it can be readily implemented thanks to its compatibility with existing pricing structures in the retail market. Extensive numerical results show that the proposed approach provides a win-win solution for both the LSE and its customers.

Index Terms—Load serving entity, bilevel programming, demand response, dynamic pricing, complementarity modeling.

NOMENCLATURE

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>DER</td>
<td>Distributed energy resource</td>
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<tr>
<td>DG</td>
<td>Dispatchable distributed generator</td>
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<td>DR</td>
<td>Demand response</td>
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<td>ILC</td>
<td>Involuntary load curtailment</td>
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<td>LSE</td>
<td>Load serving entity</td>
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<tr>
<td>PCC</td>
<td>Point of common coupling</td>
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<tr>
<td>RES</td>
<td>Renewable energy source</td>
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<td>RESSF</td>
<td>RES scaling factor</td>
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</table>

Indices

- $d$: Index of DR aggregators
- $i$: Index of DGs
- $k$: Index of batteries
- $m$: Index of demand blocks of DR aggregators
- $t$: Index of time slots

Parameters and Constants

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\Delta T$</td>
<td>Length of time slot</td>
</tr>
<tr>
<td>$\eta^d_{b,k}$</td>
<td>Charging/discharging efficiency of battery $k$</td>
</tr>
<tr>
<td>$c^L_{ILC}$</td>
<td>Cost of involuntary load curtailment ($/MWh$)</td>
</tr>
<tr>
<td>$c^R_{RES}$</td>
<td>Renewable energy cost at time $t$ ($/MWh$)</td>
</tr>
<tr>
<td>$c^R_k$</td>
<td>Regular retail price at time $t$ ($/MWh$)</td>
</tr>
<tr>
<td>$CU_{i,t}$</td>
<td>Start-up offer cost of DG $i$ ($)</td>
</tr>
<tr>
<td>$D_i$</td>
<td>Inflexible load at time $t$ (MW)</td>
</tr>
<tr>
<td>$DR_{i,t}$</td>
<td>Ramping-down/up rate limit of DG $i$ (MW)</td>
</tr>
<tr>
<td>$DT_{i,t}$</td>
<td>Minimum down/up time of DG $i$ (h)</td>
</tr>
</tbody>
</table>

$E_d$ Minimum total energy consumption of DR aggregator $d$ over the scheduling horizon (MWh)

$E_k$ Capacity of battery $k$ (MWh)

$N_D$ Number of DR aggregators

$NG, NB$ Number of DGs/batteries

$NM_d$ Number of demand blocks of DR aggregator $d$

$NT$ Number of time slots

$P_{min}^{d,m,t}, P_{max}^{d,m,t}$ Minimum/maximum power generation of DG $i$ at time $t$ (MW)

$P_{g, max}^k, P_{RES,a}^c$ Available renewable generation at time $t$ (MW)

$P_{max}^{d,m,t}$ Maximum load of demand block $m$ of DR aggregator $d$ at time $t$ (MW)

$P_{min}^{d,t}$ Minimum power consumption of DR aggregator $d$ at time $t$ (MW)

$P_{c}^k$ Maximum charging power of battery $k$ (MW)

$P_{dis}^k$ Maximum discharging power of battery $k$ (MW)

$U_{i,t}^{D}, U_{i,t}^{U}$ Ramping down limit of DR aggregator $d$ (MW)

$R_{u}, R_{d}$ Ramping up limit of DR aggregator $d$ (MW)

$U_{i,t}^{D}$ Marginal utility of demand block $m$ of DR aggregator $d$ at time $t$ ($/MWh$)

$U_{d,t}(\cdot)$ Utility function of DR aggregator $d$ at time $t$

$y_{i,t}$ Binary variable, “1” if charging/discharging

$I_{i,t}$ Production cost of DG $i$ ($)  

$C_{i,t}$ Retail price for DR aggregators ($/MWh$)  

$I_{i,t}^{DR}$ Involuntary load curtailment at time $t$ (MW)  

$P_{t}^{RES}$ Commitment status of DG $i$ at time $t$ \{0, 1\}  

$P_{t}^{U}$ Power exchange with the main grid at time $t$ (MW)  

$P_{t}^{RES}$ Scheduled renewable generation at time $t$ (MW)  

$P_{t}^{d,m,t}$ Scheduled load for demand block $m$ of DR aggregator $d$ (MW)  

$P_{t}^{d,t}$ Scheduled load for DR aggregator $d$ (MW)  

$P_{g,t}$ Power generation of DG $i$ at time $t$ (MW)  

$P_{c,t}^k$ Charging power of battery $k$ at time $t$ (MW)  

$P_{dis}^k$ Discharging power of battery $k$ at time $t$ (MW)  

$SOC_{k,t}$ State of charge of battery $k$  

$SU_{i,t}$ Start-up cost of DG $i$ ($)  

$y_{i,t}, z_{i,t}$ Start-up and shutdown indicators \{0, 1\}  

I. INTRODUCTION

Intelligent demand response (DR) is one of the most important characteristics of the smart grid. DR enables utilities and system operators to defer the upgrade of their electricity networks, reduce their operating costs, and provide customers opportunity to lower their electricity bills. Description of various DR benefits in electricity markets and power system operation can be found in [1]. Additionally, DR has been
recognized as an effective tool to facilitate the integration of intermittent and stochastic energy sources such as wind or solar energy into the power system [2], [3].

A great deal of research has been carried out to study DR at different system levels. Reference [4] proposed a stochastic security constrained unit commitment model to analyse the integration of DR sources in the wholesale electricity market. A novel price-based optimization framework for a DR aggregator was introduced in [5] to maximize the profit of the DR aggregator. The impact of DR integration on the market clearing price was investigated in [6]. In [7], the authors considered the potential of DR resources for providing the frequency regulation service. In [8]–[10], different models were proposed to maximize the benefits of large energy customers with DR capabilities. In addition, extensive research has been conducted to study residential energy management [11], [12]. Different multi-objective optimization energy management frameworks were introduced in [14], [15] to minimize the energy cost for a residential household considering customer thermal comfort preferences. A novel load shaping strategy was presented in [16] exploiting the dynamic pricing and energy storage.

This paper proposes a novel pricing design for DR in the distribution network by using the bilevel programming approach. In fact, bilevel programming has been used to study different problems in power systems such as transmission and generation expansion planning [17], [18], generation maintenance [19], market equilibria [20], and strategic bidding for power producers [21], retailers [22], and distribution companies [23]. Recently, there have been some research works on the energy management design for smart grids using bilevel programming. In particular, Asimakopoulou et al. [24] formulated a bilevel problem to study the interaction between a large central production unit and an energy service provider (ESP) managing several microgrids (MGs). The central production unit computes and sends an optimal energy price signal to the ESP, then the ESP decides the optimal amount of energy purchased from the central generation unit as well as schedules its power generation and consumption accordingly. However, renewable energy sources (RESs) and the interaction with the main grid (utility grid) were not considered in this paper. Moreover, the formulated problem is a nonlinear mixed integer problem which requires nonlinear solvers.

Stochastic bilevel formulation was also proposed in [25] to analyze the interaction between a distribution network operator (DNO) and networked MGs considering the renewable energy generation uncertainty where each entity aims at minimizing its individual operation cost. The information exchanged between the DNO and MGs includes the generation and demand of MGs while the price of energy exchange between the DNO and MGs is fixed. In both [24], [25], the authors studied single-period optimization problems. In contrast, we consider a multi-period optimization problem which is able to capture time-coupling constraints such as raming limit of dispatchable DGs, charging/discharging constraints of batteries, especially the price arbitrage potential in electricity markets.

The research objective of this work is very different from the ones in our previous works [2], [3]. In [2], we presented an optimal day-ahead price based energy management framework for a microgrid (MG) considering users’ thermal comfort requirements where the whole system is centrally controlled by the MG aggregator. The main goal of [2] is to study the benefits of thermal load as a valuable DR source in the smartgrid. The interactions between the MG aggregator and DR sources (i.e., thermal loads) as well as energy end-users were not considered in [2]. A risk-aware stochastic optimization model was proposed in [3] to maximize the profit of a MG aggregator. Although the interaction between energy customers and the MG aggregator was explicitly considered in [3], the price that the MG aggregator offers to flexible loads is predefined. Additionally, the key design output of the optimization models in both [2] and [3] is the optimal hourly bids that the MG aggregator submits to the day-ahead market to maximize its profit.

In this work, the retail price that the LSE charges flexible loads is set dynamically, which depends on actual operation conditions of the system (e.g., renewable energy generation, grid electricity price, status of batteries and DGs). The key optimization variable in this design is the DR price which is defined as the retail price that the LSE charges flexible load customers. Hence, it is expected that the DR capability of flexible loads can be exploited more efficiently to maximize the benefits of both the LSE and energy customers. Table I describes a few state-of-the-art designs related to DR research, which help demonstrate the novelty of our proposed design compared to the existing literature. In particular, our proposed system is suitable for exploiting DR capabilities of small and medium-sized customers in the distribution network while it does not require significant changes to the existing market structure. Furthermore, the proposed pricing design considers the time-varying nature of the operation conditions of system components under the control of the LSE and the willingness of changing loads from customers so that the optimal DR price will maximize the benefits of both the LSE and its customers. Finally, our pricing design takes some practical aspects of economic design in the distribution network to attract flexible load customers to participate in our scheme, as will be explained in the following.

Different from prevailing time-varying pricing schemes such as time-of-use (TOU) and real-time pricing (RTP) for retail customers [11], which may increase the energy cost for some customers with small flexible loads, the proposed scheme does not have negative impacts on inflexible customers. Our proposed model aims at exploiting flexible loads to achieve efficient operations of a LSE via a smart pricing scheme which ensures cost saving for energy customers of the LSE. Indeed, there can be various uncertainty factors in the system such as renewable energy generation and grid electricity price. However, uncertainty modeling is not considered in this paper since our main design objective is to demonstrate the benefits of smart pricing for facilitating DR integration into the distribution network. It is possible to extend our model to integrate system uncertainties, for example, by using stochastic optimization frameworks as considered in our previous works [2], [3] and other popular optimization techniques such as robust optimization [8]. Our main contributions can be summarized as follows.

- We present a comprehensive decision-making framework for short-term operation of a LSE in the future smart grids...
where distributed energy resources (DERs), renewable energy, DR, and other important system parameters are considered. We introduce a novel and practical pricing model for DR loads in the distribution network. The proposed model can be readily implemented since it does not require any significant changes to the existing retail market structure.

- We model the interaction between the LSE and its customers as a bilevel programming problem where the LSE is the leader and each DR aggregator is a follower. The nonlinear bilevel mixed-integer program is transformed into a single mixed integer linear program (MILP) using some transformation techniques such as the Karush-Kuhn-Tucker (KKT) optimality conditions and strong duality theorem. The outcome of this problem contains the optimal hourly retail prices for flexible (DR) loads. Extensive numerical results show that the proposed scheme provides a win-win solution for both the LSE and its customers. In particular, it can help improve the optimal profit for the LSE, increase the payoffs for DR aggregators, and decrease the amount of potential involuntary load curtailment as well as renewable energy curtailment.

The remaining of this paper is organized as follows. In Section II, we describe the proposed system model. Section III formulates the problem and Section IV presents the solution approach. Numerical results are shown in Section V followed by conclusion in Section VI.

### II. System Model

The energy scheduling problem is considered in a one-day period which is divided into 24 equal time slots. We consider a LSE which can procure energy from various sources including the main grid, DR resources, batteries, and local DERs including RESs (e.g., wind and solar energy) and dispatchable DGs (e.g., diesel generators, microturbines, and fuel cells) to serve its customers. Fig. 1 illustrates the considered system model.

The LSE itself may possess some DERs and it can also buy energy from privately owned DERs (e.g., from third party companies, households). If the LSE purchases electricity from third party companies or households, it must pay these entities for the procured energy. The price paid to each privately owned DER can be different, which depends on specific agreements or contracts between the LSE and those sources. If a third party company or a household owns some DERs, the company or the household is responsible for the operation cost of those energy generating sources; however, it can receive the revenue from selling energy to the LSE. On the other hand, if the

<table>
<thead>
<tr>
<th>Paper</th>
<th>Solution approach</th>
<th>Pros</th>
<th>Cons</th>
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<tbody>
<tr>
<td>[2]</td>
<td>Stochastic programming</td>
<td>-Optimal day-ahead energy bidding design</td>
<td>-Pricing design is not addressed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Utilize building thermal mass as a DR resource</td>
<td>-Interaction between the MG aggregator and DR sources is not considered explicitly</td>
</tr>
<tr>
<td>[3]</td>
<td>Stochastic programming</td>
<td>-Optimal day-ahead energy bidding design</td>
<td>-DR prices in DR contracts are fixed and do not adapt as well as depend on the actual system operation conditions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Risk measure is considered vs DR resources via economic contracts</td>
<td>-DR aggregators interact directly with the wholesale market, which complicates the wholesale market operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Interaction between MG aggregator and medium-sized customers are considered, including load curtailment, load shifting, utilizing on-site generation, and utilizing energy storage</td>
<td>-Requires significant changes in the wholesale market operation</td>
</tr>
<tr>
<td>[5]</td>
<td>MILP</td>
<td>-Various DR procurement strategies from small and medium-sized customers are considered</td>
<td>-Requires significant changes in the wholesale market to integrate DR bids</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Including load curtailment, load shifting, utilizing on-site generation, and utilizing energy storage</td>
<td></td>
</tr>
<tr>
<td>[8]</td>
<td>Rolling robust optimization</td>
<td>-Maximize utility of energy customers (households or small businesses) considering price uncertainty</td>
<td>-Customers are passive entities who receive prices given by system operators and maximize their utility accordingly</td>
</tr>
<tr>
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<td>-DR aggregators interact directly with the wholesale market, which complicates the wholesale market operation</td>
<td>-Pricing design is not considered</td>
</tr>
<tr>
<td>[9]</td>
<td>Rolling robust optimization</td>
<td>-Maximize utility of energy customers (microgrids, virtual power plants) considering price and renewable energy uncertainties</td>
<td>-DR aggregators interact directly with the wholesale market, which complicates the wholesale market operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Optimal hourly retail prices for flexible (DR) loads</td>
<td>-Pricing design between system controller and flexible loads is not considered</td>
</tr>
<tr>
<td>[10]</td>
<td>Stochastic bilevel programming</td>
<td>-Optimize the bidding curve of a large customer in the pool market considering uncertainties</td>
<td>-The model targets large customers</td>
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<tr>
<td></td>
<td></td>
<td>-Large customers trade directly in pool market</td>
<td>-Does not consider DR integration in the distribution network level</td>
</tr>
<tr>
<td>[11]</td>
<td>Rolling stochastic programming and robust optimization</td>
<td>-Motivates load shifting of residential load based on real time pricing signal</td>
<td>-Pricing signal design is not mentioned</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Detailed modeling of each residential appliances</td>
<td>-Negative impact on less flexible customers</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Applicable for small scale residential load</td>
<td></td>
</tr>
<tr>
<td>[14], [15]</td>
<td>Multi-objective optimization</td>
<td>-Comfort and lifestyle are addressed</td>
<td>-Applicable for small scale residential load</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Maximize comfort and lifestyle are addressed</td>
<td>-Pricing signal design is not mentioned</td>
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<tr>
<td>[16]</td>
<td>Distributed Optimization</td>
<td>-Maximize customer’s utility</td>
<td>-The final problem is a MILP</td>
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<td></td>
<td></td>
<td>-Minimize grid fluctuation</td>
<td>-Single period optimization</td>
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<tr>
<td></td>
<td></td>
<td>-Dynamic pricing design for households</td>
<td>-Does not consider renewable energy and main grid</td>
</tr>
<tr>
<td>[24]</td>
<td>Bilevel programming</td>
<td>-Pricing design between microgrids and a LSE</td>
<td>-Pricing design among MGs is not considered</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Cost minimization of individual MG is addressed</td>
<td>-Pricing signals among MGs should be optimized</td>
</tr>
<tr>
<td>[25]</td>
<td>Stochastic bilevel programming</td>
<td>-Uncertainties are captured</td>
<td>-Pricing design among MGs is not considered</td>
</tr>
<tr>
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<td></td>
<td>-Cost minimization of individual MG is addressed</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>-Applicable for networked MGs</td>
<td></td>
</tr>
<tr>
<td>Our paper</td>
<td>programming</td>
<td>-A novel and practical pricing scheme between a LSE and energy customers</td>
<td>-Uncertainties are not considered in this paper and will be the subject of our future work</td>
</tr>
</tbody>
</table>

where distributed energy resources (DERs), renewable energy, DR, and other important system parameters are considered. We introduce a novel and practical pricing model for DR loads in the distribution network. The proposed model can be readily implemented since it does not require any significant changes to the existing retail market structure.

- We model the interaction between the LSE and its customers as a bilevel programming problem where the LSE is the leader and each DR aggregator is a follower. The nonlinear bilevel mixed-integer program is transformed into a single mixed integer linear program (MILP) using some transformation techniques such as the Karush-Kuhn-Tucker (KKT) optimality conditions and strong duality theorem. The outcome of this problem contains the optimal hourly retail prices for flexible (DR) loads. Extensive numerical results show that the proposed scheme provides a win-win solution for both the LSE and its customers. In particular, it can help improve the optimal profit for the LSE, increase the payoffs for DR aggregators, and decrease the amount of potential involuntary load curtailment as well as renewable energy curtailment.

The remaining of this paper is organized as follows. In
The LSE operates some DERs by itself, the operation cost of those DERs is imposed directly on the LSE.

For simplicity, we assume that the LSE possesses several conventional DGs such as diesel generators and fuel cells, and it does not buy energy from privately owned conventional DGs. Additionally, the LSE does not own any renewable energy sources. We assume that the LSE has take-or-pay contracts [9], which are also called Power Purchase Agreements (PPA) in some markets [3], [9], with local wind farms and/or solar farms to buy renewable energy from them. In the take-or-pay contracts, the LSE buys all available renewable energy generated from these wind/solar farms at a fixed price which is typically lower than the average price from the main grid [9]. Without loss of generality, we assume that the prices paid to all renewable energy sources are the same \( c^\text{RES}_i \). Finally, the LSE may own some battery storage units.

System loads are assumed to belong to one of the two categories, namely flexible and inflexible loads. Inflexible loads or critical loads are those that the LSE has to serve. If the LSE cannot simply serve the inflexible loads, a portion of the inflexible loads has to be shed, which is called involuntary load curtailment (ILC). A very high penalty cost \( c^\text{LC} \) is imposed on the LSE for ILC since the main goal of the LSE is to guarantee electricity supply to its customers [2]. Inflexible loads are charged under the regular retail price \( c^\text{P}_b \). In contrast, flexible loads are assumed to be aggregated by one or several DR aggregators which enjoy a dynamic pricing tariff that should be designed to bring advantages to the DR aggregators. One practical strategy to encourage DR aggregators participating in our proposed operation model is to ensure cost saving for them.

In practice, a flexible load customer might be hesitant to participate in a real-time pricing scheme since electricity prices in this scheme may be greater than the regular retail price for several hours of a day. The loads of a flexible load customer include critical load which should not be shed or shifted and flexible load that can be shed or shifted. Therefore, if the flexible load customer has a large portion of critical load during high price hours, we might not be able to guarantee cost saving for the customer compared to the case where the customer is charged at the fixed retail price. Hence, one of the most practical approaches that the LSE may use to attract flexible load customers to participate in the proposed pricing model is to offer DR price (i.e., the retail price that the LSE charges flexible loads or DR aggregators), which is always lower or equal to the retail price in each hour. In the worst case when the DR price is equal to the regular retail price, the cost imposed on participating entities is the same with the one when they are charged under the regular retail price.

The proposed system model can be applied to the practical setting where a LSE provides energy services to a certain geographical area. In particular, there can be several DR aggregators in the area which aggregate flexible loads from energy users and each DR aggregator serves a given set of flexible loads. A DR aggregator can be a company which is interested in the DR market (e.g., EnerNOC). This design allows us to prevent individual small flexible energy customers from interacting directly with the wholesale market, which would complicate the operation of the wholesale market. Moreover, our design ensures that the number of participating parties in our model as well as the number of variables in our formulated optimization problem be reduced significantly. In addition, we assume that DR aggregators have DR contracts with flexible load customers so that these customers can declare the characteristics of their loads (e.g., utility function [9], [10], [28]–[30] or discomfort function in the case of load reduction or load shifting [3], [5], [10]) to the DR aggregators. Based on the load information provided by their customers, each DR aggregator can construct a suitable aggregated utility function, as described in Section III-I, which is then sent to the LSE. Detailed study on how DR aggregators interact with their customers and construct their aggregated utility function is out of scope of this paper.

The underlying optimization problem is formulated as a bilevel program where the LSE is the leader and each DR aggregator is a follower. The outcome of this problem contains optimal dynamic DR price series \( c^\text{DR}_i \) over the scheduling horizon. Additionally, the outputs of the proposed problem include the hourly energy trading between the LSE and the main grid \( P^\text{P}_g \), the scheduled generation of local RESs \( P^\text{RES}_i \) and local DGs \( P^\text{DG}_i \), charging/discharging power of batteries \( P^\text{batt}_k \), amount of ILC \( D^\text{LC}_t \), and hourly energy consumption of DR aggregators \( P^\text{d}_t \).

### III. Problem Formulation

#### A. Objective Function of the LSE

We are interested in maximizing the profit of the LSE which is given as follows:

\[
\text{Profit} = \text{Rev} - \text{Cost}
\]  

\[\text{http://www.enernoc.com/} \]
where \( Rev \) is the retail revenue obtained by serving inflexible loads (at price \( c^R_t \)) and flexible loads (at price \( c^{DR}_t \)), i.e.,
\[
Rev = \sum_{t=1}^{NT} \Delta T \left[ c^R_t (D_t - D^C_t) + \sum_{d=1}^{ND} c^{DR}_t P_{d,t} \right] \tag{2}
\]
where \( D_t - D^C_t \) is the amount of inflexible load that the LSE serves at time \( t \).

The operating cost of the LSE includes the cost of buying/selling electricity from/to the main grid, renewable energy procurement cost, operation costs of DGs including start-up and dispatch cost [3], and the penalty cost for involuntary load curtailment. For simplicity, the battery operation cost is not considered in this paper. Hence, we have
\[
\text{Cost} = \sum_{i=1}^{NT} \Delta T \left[ P^e_t c^e_t + c^{RES}_t P^{RES,a}_t \right. \\
+ \left. \sum_{i=1}^{NG} (SU_{i,t} + C_i(P_{i,t})) + c^L_t D^C_t \right]. \tag{3}
\]

From these expressions, the design objective becomes
\[
\text{maximize} \quad c^e_t, P^e_t, P^{RES,a}_t, P^{RES}_t, P^d_t, P_{d,t} \sum_{t=1}^{NT} \Delta T \left[ c^R_t (D_t - D^C_t) \right. \\
+ \left. \sum_{d=1}^{ND} c^{DR}_t P_{d,t} \right] - \sum_{t=1}^{NT} \Delta T \left[ P^e_t c^e_t + c^{RES}_t P^{RES,a}_t \right. \\
+ \left. \sum_{i=1}^{NG} (SU_{i,t} + C_i(P_{i,t})) + c^L_t D^C_t \right] \tag{4}
\]
subject to the following constraints.

B. Power Balance Constraints
\[
P^e_t + \sum_{i=1}^{NB} P_{i,t} + P^{RES}_t + \sum_{k=1}^{NB} (P^d_{k,t} - P^e_{k,t}) + D^C_t = D_t + \sum_{d=1}^{ND} P_{d,t}, \quad \forall \ t. \tag{5}
\]

C. Power Trading with Main Grid
\[
-P^\text{max}_t \leq P^e_t \leq P^\text{max}_t, \quad \forall \ t. \tag{6}
\]

D. Renewable Energy Constraints
The scheduled renewable energy generation must be smaller or equal to the available renewable energy generation. Hence,
\[
0 \leq P^{RES}_t \leq P^{RES,a}_t, \quad \forall \ t. \tag{7}
\]

E. Involuntary Load Curtailment
\[
0 \leq D^C_t \leq D_t, \quad \forall \ t. \tag{8}
\]

F. DR Price
As explained in Section II, the DR price is set to be smaller or equal to the regular retail price at every time slot \( t \) to attract the participation of DR aggregators in the proposed DR pricing scheme, i.e., we have
\[
c^{DR}_t \leq c^R_t, \quad \forall \ t. \tag{9}
\]

G. Operation Constraints of DGs
In this paper, a widely used piecewise linear cost function [2], [33], [34] is employed to model approximately the production cost \( C_i(\cdot) \) of DG \( i \) where \( n \) and \( N_t \) are the segment indices and number of segments in the cost function of DG \( i \), respectively. Parameter \( \lambda_i \) ($/MWh) denotes the marginal cost associated with segment \( n \) in the cost function of DG \( i \). The cost of operating DG \( i \) at its minimum power generation [33] is \( a_i \). Finally, we define \( P_{i,n} \) (MW) as the upper limit of power generation from the \( n \)-th segment in the cost function of DG \( i \) and \( P_{i,n,t} \) is scheduled power generation of DG \( i \) from the \( n \)-th segment at time \( t \). We have [2], [33], [34]
\[
C_i(P_{i,t}) = a_i I_{i,t} + \Delta T \sum_{n=1}^{N_t} \lambda_i P_{i,n,t} \tag{10}
\]
\[
0 \leq P_{i,n,t} \leq P_{i,n}; \quad P_{i,t} = P_{i}^\text{min} I_{i,t} + \sum_{n=1}^{N_t} P_{i,n,t} \tag{11}
\]
where constraints (10)–(11) describe the generation cost and power output of DG \( i \). Additionally, the following constraints are imposed on the operation of DG \( i \) [34]:
\[
P_{i}^\text{min} I_{i,t} \leq P_{i,t} \leq P_{i}^\text{max} I_{i,t}; \quad SU_{i,t} = CU_i y_{i,t} \tag{12}
\]
\[
P_{i,t} - P_{i,t-1} \leq U R_i \tag{13}
\]
\[
P_{i,t-1} - P_{i,t} \leq D R_i \tag{14}
\]
\[
\sum_{h=t}^{t+D T_i - 1} I_{i,h} \geq U T_i y_{i,t} \tag{15}
\]
\[
\sum_{h=t}^{t+D T_i - 1} (1 - I_{i,h}) \geq D T_i z_{i,t} \tag{16}
\]
\[
y_{i,t} - z_{i,t} = I_{i,t} - I_{i,t-1}; \quad y_{i,t} + z_{i,t} \leq 1. \tag{17}
\]

Power generation limits and start-up cost are described in (12). The remaining constraints capture ramping up/down limits, limits on minimum ON/OFF duration, and relationship between binary variables [3], [33], [34].

H. Battery Constraints
The following constraints are imposed on the operation of battery \( k \) for \( \forall t \) [2], [3], [35]:
\[
0 \leq P^e_{k,t} \leq b^e_{k,t}; \quad 0 \leq P^d_{k,t} \leq b^d_{k,t} P^e_{k,t} \tag{18}
\]
\[
SOC_{k,t}^\text{min} \leq SOC_{k,t} \leq SOC_{k,t}^\text{max} \tag{19}
\]
\[
SOC_{k,t+1} = SOC_{k,t} + \Delta T \left( P^e_{k,t} - P^d_{k,t} \right) \tag{20}
\]
\[
b^e_{k,t} + b^d_{k,t} \leq 1; \quad b^e_{k,t}, b^d_{k,t} \in \{0, 1\}. \tag{21}
\]

Limits on the charging and discharging power of battery \( k \) are presented in (18). Constraint (19) imposes limits on State of Charge (SOC) of battery \( k \). Note that \( SOC_{k,t}^\text{max} \) and \( SOC_{k,t}^\text{min} \) are the maximum SOC and minimum SOC of battery \( k \), respectively. Battery energy dynamics model is given in (20). Finally, conditions on binary variables representing battery charging/discharging status are captured in (21) so that battery \( k \) cannot be charged and discharged simultaneously.
In addition to the above operation constraints, the optimization of the leader problem is subject to $ND$ follower problems each of which corresponds to an optimization problem of DR aggregator $d$. The lower-level problem for DR aggregator $d$ is presented in the following. First, we define the payoff function for each DR aggregator as the utility (benefit) minus the cost due to energy consumption over the scheduling horizon. We assume that each DR aggregator wishes to maximize its payoff function as follows:

$$\text{maximize } \sum_{t=1}^{NT} \left[ U_{d,t}(P_{d,t}) - \Delta T c_t^{DR} P_{d,t} \right].$$  (22)

In this paper, the utility functions of DR aggregators are modeled by multi-block utility functions\(^3\), which are commonly used in the literature [9], [10], [28], [29]. The marginal utility of a demand block decreases as the index of demand blocks increases. Fig. 2 shows the utility function of DR aggregator $d$ at time $t$. As we can observe, this function has four demand blocks (i.e., $NMD = 4$). The values at point A, C, D, E are $p_{d,1}^{max}$, $p_{d,2}^{max}$, $p_{d,3}^{max}$, and $p_{d,4}^{max}$, respectively. If the scheduled demand of DR aggregator $d$ at time $t$ is $OB$ (i.e., $P_{d,t} = OB$), then the utility value for load consumption of aggregator $d$ at time $t$ is equal to the shaded area. Generally, we have

$$U_{d,t}(P_{d,t}) = \Delta T \sum_{m=1}^{NMD} u_{d,m,t} P_{d,m,t}$$  (23)

$$P_{d,t} = \sum_{m=1}^{NMD} P_{d,m,t}.$$  (24)

Therefore, the follower (lower) optimization problem of DR aggregator $d$ can be written as follows:

$$\text{minimize } \Delta T \sum_{t=1}^{NT} c_t^{DR} P_{d,t} - \sum_{m=1}^{NMD} u_{d,m,t} P_{d,m,t}.$$  (25)

The power constraints for each demand block $m$ for the flexible load of DR aggregator $d$ are captured in (27)-(28). Constraint (29) describes the minimum energy consumption for the load of DR aggregator $d$ over the scheduling horizon. The constraint on the minimum power consumption for DR aggregator $d$ at each time slot $t$ is expressed in (30) while maximum power consumption constraints for DR aggregator $d$ are described in (26)-(27). Finally, (31)-(32) impose the ramping up and ramping down constraints for the load of DR aggregator $d$ where $P_{d,0}$ is the initial load of DR aggregator $d$.

\(^3\)In the literature, there exist other models for flexible loads. For example, price elasticity of the load model is considered in [49]–[51]. However, we choose the multi-block utility function model since it is suitable for the proposed solution approach and current practice in the electricity market.

**J. Extension with Power Flow Constraints**

For ease of exposition, in the problem formulation described above, we have implicitly assumed that all entities are located at one bus, which is valid for a small-scale system (e.g., a LSE manages loads in a small town or a village). However, a general distribution network model can also be integrated into our optimization framework. The power flow constraints are described as follows. We define $p, q$ as the indices of two buses, $B_{p,q}$ is the susceptance of line $p-q$, $F_{p,q}^{max}$ is the transmission capacity of line $p-q$, and $\theta_{p,t}$ is the voltage angle of bus $p$ at time $t$. Additionally, we define $A_p$ as the set of buses connected to bus $p$, $B_p$ as the set of batteries located at bus $p$, $C_p$ is the set of DGs located at bus $p$, and $D_p$ as the set of DR aggregators located at bus $p$. Moreover, $D_{p,t}$ is the total inflexible load at bus $p$ and time $t$ while $D_{p,t}^{LC}$ is the amount of involuntary load curtailment at bus $p$ and time $t$ ($D_{p,t}^{LC} \leq D_{p,t}$ for all $p, t$). In addition, $P_{RES}^{p,t}$ is the amount of scheduled renewable energy generation at bus $p$ and time $t$, and $P_{E}^{p,t}$ is the amount of energy exchange with the main grid at bus $p$ and time $t$. Note that $P_{E}^{p,t} = 0$, for all $p$ if bus $p$ is not connected to the main grid.

For simplicity, reactive power is not considered in this paper and the lossless DC power flow model is used to model the distribution network [45], which imposes the following constraints:

$$P_{E}^{p,t} + \sum_{i \in C_p} P_{i,t} + P_{RES}^{p,t} + \sum_{k \in B_p} (P_{k,t}^{d} - P_{k,t}^{e}) + D_{p,t}^{LC} \leq D_{p,t}$$  (33)

$$-D_{p,t} + \sum_{d \in D_p} P_{d,t} = \sum_{q \in A_p} B_{p,q}(\theta_{p,t} - \theta_{q,t}), \forall p, t.$$  (34)

The values at point A, C, D, E are $p_{d,1}^{max}$, $p_{d,2}^{max}$, $p_{d,3}^{max}$, and $p_{d,4}^{max}$, respectively. If the scheduled demand of DR aggregator $d$ at time $t$ is $OB$ (i.e., $P_{d,t} = OB$), then the utility value for load consumption of aggregator $d$ at time $t$ is equal to the shaded area. Generally, we have
where constraint (33) enforces the power balance at each bus in the system while constraint (34) presents power flow limits of each line. All operation constraints of the LSE as well as the follower problem remain the same as in Section III. However, the power balance constraint (5) is replaced by the set of power flow constraints presented above.

We are aware of the fact that the lossless DC power flow model may not be the most suitable for the distribution network. Integration of a full power flow model and reactive power management into the proposed optimization framework is a subject of our future work.

IV. Solution Approach

We propose to convert the optimization problem of the LSE into an equivalent MILP problem. Note that optimization variables in each follower problem include \( P_{d,m,t} \) and \( P_{d,t} \). Moreover, the variable \( c^{\text{DR}} \) in the upper-level (leader) problem is a parameter in each lower-level (follower) problem. Also, for a given vector \( c^{\text{DR}} \), each follower problem is simply a linear program. Therefore, we can replace each follower problem with its corresponding KKT optimality conditions [22]. Toward this end, the Lagrangian of each lower-level problem (25)-(32) for DR aggregator \( d \) can be expressed as

\[
L_d = \Delta T \sum_{t=1}^{NT} \left[ c^{\text{DR}}_t P_{d,t} - \sum_{m=1}^{NM_d} u_{d,m,t} P_{d,m,t} \right] + \sum_{t=1}^{NT} \lambda_{d,t} (P_{d,t} - \sum_{m=1}^{NM_d} P_{d,m,t}) + \sum_{t=1}^{NT} \sum_{m=1}^{NM_d} \mu_{d,m,t} (P_{d,m,t} - P_{d,m,t}^{\text{max}}) - \sum_{t=1}^{NT} \sum_{m=1}^{NM_d} \mu_{d,m,t}^2 P_{d,m,t} - \frac{3}{2} (\Delta T \sum_{t=1}^{NT} P_{d,t} - E_d)
\]

where \( \lambda_{d,t}, \mu_{d,m,t}, \mu_{d,m,t}^2, \mu_{d,m,t}^3, \mu_{d,t}^4, \mu_{d,t}^5, \mu_{d,t}^6 \), and \( \mu_{d,t}^6 \) denote the Lagrange multipliers associated with the constraints in the corresponding follower problem. The KKT necessary optimality conditions of the lower-level problem of DR aggregator \( d \) include the primal feasibility constraint (26) and the following constraints

\[
\frac{\delta L_d}{\delta P_{d,t}} = \Delta T c^{\text{DR}}_t + \lambda_{d,t} - \Delta T \mu_{d,t}^3 - \mu_{d,t}^4 + \mu_{d,t}^5 - \mu_{d,t-1}^6 = 0, \quad \forall t < NT
\]

\[
\frac{\delta L_d}{\delta P_{d,m,t}} = \Delta T u_{d,m,t} - \lambda_{d,t} + \mu_{d,m,t}^4 - \mu_{d,m,t}^5 = 0, \quad \forall m, t
\]

\[
0 \leq \mu_{d,m,t}^1 P_{d,m,t}^{\text{max}} - P_{d,m,t} \geq 0, \quad \forall m, t
\]

\[
0 \leq \mu_{d,m,t}^2 P_{d,m,t} \geq 0, \quad \forall m, t
\]

\[
0 \leq \mu_{d,t}^3 \Delta T \sum_{t=1}^{NT} P_{d,t} - E_d \geq 0,
\]

\[
0 \leq \mu_{d,t}^4 P_{d,t} - P_{d,t}^{\text{min}} \geq 0, \quad \forall t
\]

\[
0 \leq \mu_{d,t}^5 R_{d}^{\text{U}} - P_{d,t} + P_{d,t-1} \geq 0, \quad \forall t
\]

\[
0 \leq \mu_{d,t}^6 R_{d}^{\text{D}} - P_{d,t-1} + P_{d,t} \geq 0, \quad \forall t.
\]

Complementarity conditions associated with the inequality constraints (27)-(32) are given in (39)-(44). Note that a complementarity condition \( 0 \leq \mu \perp P \geq 0 \) (i.e., \( P \geq 0; \mu \geq 0 \)) can be transformed into the following set of mixed-integer constraints based on the Fortuny-Amat transformation [10], [18], [36]:

\[
\mu \geq 0; \quad P \geq 0
\]

\[
\mu \leq (1-u)M
\]

\[
P \leq uM
\]

\[
u \in \{0,1\}
\]

where \( M \) is a sufficiently large constant. Note that the value of \( M \) will affect the effectiveness of the proposed solution. In particular, we should select \( M \) appropriately to avoid numerical ill-conditioning [37]. Several guidelines on how to select a suitable value of \( M \) can be found in [36]-[38]. We need to select a sufficiently large value of \( M \) so as not to make the optimal solution outside the feasible space of (46) [37]. On the other hand, a too large value of \( M \) may result in computational inefficiencies for the solution of the resulting mixed-integer optimization problems [37]. A general principle to find a reasonable constant \( M \) is based on the trial and error approach [38]. However, in some cases, a suitable value of \( M \) can be found based on specific characteristics of the studied problems [38].

Therefore, the set of constraints (39)-(44) can be rewritten as follows:

\[
\mu_{d,m,t}^1 \geq 0; \quad P_{d,m,t}^{\text{max}} - P_{d,m,t} \geq 0
\]

\[
\mu_{d,m,t}^1 \leq (1-v_{d,m,t}^1) M^1; \quad P_{d,m,t}^{\text{max}} - P_{d,m,t} \leq v_{d,m,t}^1 M^{1}
\]

\[
\mu_{d,m,t}^2 \geq 0; \quad P_{d,m,t} \geq 0
\]

\[
\mu_{d,m,t}^2 \leq (1-v_{d,m,t}^2) M^2; \quad P_{d,m,t} \leq v_{d,m,t}^2 M^{2}
\]

\[
\mu_{d,t}^3 \geq 0; \quad \sum_{t=1}^{NT} P_{d,t} - E_d \geq 0
\]

\[
\mu_{d,t}^3 \leq (1-v_{d,t}^3) M^3; \quad \Delta T \sum_{t=1}^{NT} P_{d,t} - E_d \leq v_{d,t}^3 M^{3}
\]

\[
\mu_{d,t}^4 \geq 0; \quad P_{d,t} - P_{d,t}^{\text{min}} \geq 0
\]

\[
\mu_{d,t}^4 \leq (1-v_{d,t}^4) M^4; \quad P_{d,t} - P_{d,t}^{\text{min}} \leq v_{d,t}^4 M^{4}
\]

\[
\mu_{d,t}^5 \geq 0; \quad R_{d}^{\text{U}} - P_{d,t} + P_{d,t-1} \geq 0
\]

\[
\mu_{d,t}^5 \leq (1-v_{d,t}^5) M^5; \quad R_{d}^{\text{U}} - P_{d,t} + P_{d,t-1} \leq v_{d,t}^5 M^{5}
\]

\[
\mu_{d,t}^6 \geq 0; \quad R_{d}^{\text{D}} - P_{d,t-1} + P_{d,t} \geq 0
\]

\[
\mu_{d,t}^6 \leq (1-v_{d,t}^6) M^6; \quad R_{d}^{\text{D}} - P_{d,t-1} + P_{d,t} \leq v_{d,t}^6 M^{6}
\]

where \( M^1, M^2, M^3, M^4, M^5, \) and \( M^6 \) are sufficiently large numbers. After the follower problems are replaced
by the sets of mixed-integer linear constraints as presented above, the upper-level optimization problem is still a mixed-integer nonlinear (MILP) problem since the term \(\Delta T \sum_{t=1}^{NT} c_{t}^{D} P_{d,t} \) in the objective function (4), which is the sum of several bilinear product of variables \( c_{t}^{D} P_{d,t} \), is nonlinear. However, each term \(\Delta T \sum_{t=1}^{NT} c_{t}^{D} P_{d,t} \) of the sum can be equivalently replaced by linear expressions by using the strong duality theorem \([10],[22]\). Please refer \([48]\) for more details. The strong duality theorems renders

\[
\Delta T \sum_{t=1}^{NT} c_{t}^{D} P_{d,t} = \Delta T \sum_{t=1}^{NT} \sum_{m=1}^{NM_{d}} u_{d,m,t} P_{d,m,t} \\
+ \sum_{t=1}^{NT} \left[ - \sum_{m=1}^{NM_{d}} \mu_{d,m,t}^{1} P_{d,m,t}^{\text{max}} + \mu_{d,t}^{4} P_{d,t}^{\text{min}} + \mu_{d}^{3} E_{d} \right] \\
- \sum_{t=1}^{NT} \mu_{d,t}^{5} \bar{R}_{d}^{L} - \sum_{t=1}^{NT} \mu_{d,t}^{6} R_{d}^{D}. \tag{62}
\]

Substitute the results in (62) into the objective function in (4), we arrive at

\[
\text{Profit} = \sum_{d=1}^{ND} \Delta T \sum_{t=1}^{NT} \sum_{m=1}^{NM_{d}} u_{d,m,t} P_{d,m,t} \\
+ \sum_{t=1}^{NT} \left[ - \sum_{m=1}^{NM_{d}} \mu_{d,m,t}^{1} P_{d,m,t}^{\text{max}} + \mu_{d,t}^{4} P_{d,t}^{\text{min}} + \mu_{d}^{3} E_{d} \right] \\
- \sum_{t=1}^{NT} \mu_{d,t}^{5} \bar{R}_{d}^{L} - \sum_{t=1}^{NT} \mu_{d,t}^{6} R_{d}^{D} + \sum_{t=1}^{NT} \Delta T \left( D_{t} - D_{t}^{L} \right) \\
- \sum_{t=1}^{NT} \Delta T \left[ P_{t}^{G} c_{t}^{G} + c_{t}^{R} P_{t}^{R} \right] \\
+ \sum_{i=1}^{NG} \left( S U_{i,t} + C_{i}(P_{i,t}) \right) + c_{t}^{L} D_{t}^{L}. \tag{63}
\]

Finally, the original nonlinear bilevel optimization problem can be recast by the following MILP

\[
\begin{align*}
\text{minimize} & \quad \text{Profit} \\
\text{subject to} & \quad (5) - (9), (26), (36) - (38), (49) - (61), (62), (63).
\end{align*} \tag{64}
\]

Fig. 3 summarizes the proposed solution technique. The proposed optimization model is indeed a bilevel optimization problem, which is transformed to a single level optimization problem by replacing the lower problem with its equilibrium KKT conditions since the lower problem is linear and convex. The obtained mathematical problem with equilibrium constraints (MPEC) is, however, still difficult to solve, due to the bi-linear term and complementary constraints. Hence, strong duality theorem and big M approximation are utilized to transform the given MPEC to an equilibrium mixed integer linear programming (MILP), which can be solved efficiently by branch and bound algorithm implemented in commercial software such as CPLEX [39].

In practice, the proposed system can be implemented as follows. First, each DR aggregator collects load preference information from flexible load customers and constructs aggregated utility functions for each hour in the operating day. Then, DR aggregators send these constructed utility functions to the LSE. Based on data supplied by the forecasting entities, data related to specifications and status of batteries, DGs, and operation constraints (e.g., maximum power exchange with the main grid), and flexible load data, the LSE solves the optimization problem (59). The outcome of this problem is the optimal DR price and scheduled decisions of flexible loads. These results are sent back to the DR aggregators to implement corresponding load scheduling actions and to be utilized for quantification of cost and revenue.

\section{V. Numerical Results}

\subsection{A. Simulation Data}

Simulation data in the base case is given in Table II. Specifically, the penalty cost for involuntary load curtailment is set equal to 1000 $/MWh [2]. The renewable energy price that the LSE pays for local wind/solar farms is assumed to be 40 $/MWh. For simplicity, we assume that \( P_{t}^{G,\text{max}} = P_{t}^{\text{grid}} \) and \( c_{t}^{R} = c_{t}^{L}, \forall t \). The regular retail price in the base case is 60$/MWh and we assume the LSE does not possess any battery storage unit in the base case. Dispatchable DG data is taken from [2]. Moreover, we assume that the LSE can predict electricity price, inflexible load, and renewable energy generation with high accuracy. For simplicity, we use historical data of the corresponding system parameters as their forecast values. Electricity price data is taken from PJM website [2]. Hourly inflexible load data is retrieved from [40]. Renewable energy generation data is constructed from data in [2]. Figs. 4(a), 4(b) shows forecast data of electricity price, inflexible load, and renewable energy generation.

We assume that flexible loads are aggregated by three DR aggregators. The modeling method in [9], [10] is employed to construct flexible load data. The data of the base-case multi-block utility functions for DR aggregators is given in

\begin{center}
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Mathematical Program with Equilibrium Constraints (MPEC)} & \\
\hline
\textbf{LSE} & \\
\hline
\textbf{Maximize profit (4)} & \\
\textbf{Determine:} & \\
\hline
\text{Optimal energy exchange with grid} \thickspace \checkmark & \\
\text{Optimal exchange of local RESs, DGs, BEs and involuntary load curtailment} \thickspace \checkmark & \\
\text{LSE price:} \thickspace 60$/MWh & \\
\hline
\textbf{DR Aggregator} & \\
\hline
\text{Maximize Payoff Function (22)} & \\
\text{Determine:} & \\
\hline
\text{Load scheduling} & \\
\hline
\end{tabular}
\end{center}

Fig. 3. Summary of the proposed solution algorithm.

In the base case, the LSE is assumed to be able to handle 1000 MW of renewable energy. The renewable energy is distributed among three DR aggregators, each with a capacity of 333 MW. The DR aggregators are responsible for meeting the demand of flexible loads, which are aggregated by three DR aggregators. Each DR aggregator is responsible for a specific hour in the operating day. For simplicity, we assume that all flexible loads are aggregated by three DR aggregators. The modeling method in [9], [10] is employed to construct flexible load data. The data of the base-case multi-block utility functions for DR aggregators is given in

\begin{align*}
\text{minimize} & \quad \text{Profit} \\
\text{subject to} & \quad (5) - (9), (26), (36) - (38), (49) - (61), (62), (63).
\end{align*} \tag{64}
be zero (i.e., hourly power consumption of every DR aggregator is set to 0.6x(1+1+2+2)x24 = 86.4 MWh). For simplicity, the minimum energy consumption \((E_d)\) of each DR aggregator \(d\) over the considering day is set equal to 60% of its maximum energy consumption level (e.g., \(E_3 = 0.6x(1+1+2+2)x24 = 86.4 \text{ MWh}\)). For simplicity, the minimum hourly power consumption of every DR aggregator is set to be zero (i.e., \(p_{\text{min}} = 0, \forall d, t\)). Without loss of generality, limits on load ramping up and ramping down are assumed to be sufficiently large.

### Sensitivity Analysis

We consider the two following schemes.
- **Scheme 1 (S1):** The LSE solves the proposed optimization model. The DR aggregators enjoy a dynamic retail price tariff.
- **Scheme 2 (S2):** The LSE solves the same optimization problem. However, the regular retail price is applied to DR aggregators (i.e., \(c_d^R = R_t, \forall t\)). In this scheme, DR aggregators have no incentives to modify their loads.

### Table IV

<table>
<thead>
<tr>
<th>(c_d^R) ($/MWh)</th>
<th>Payoff 1 ($)</th>
<th>Profit 1 ($)</th>
<th>DR1 MWh</th>
<th>DR2 MWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>2607.2</td>
<td>2403.2</td>
<td>695.7</td>
<td>146.8</td>
</tr>
<tr>
<td>50</td>
<td>2061.2</td>
<td>1786.6</td>
<td>2103.9</td>
<td>1476.9</td>
</tr>
<tr>
<td>55</td>
<td>1250.2</td>
<td>778.6</td>
<td>4599.9</td>
<td>3942.8</td>
</tr>
<tr>
<td>60</td>
<td>251.0</td>
<td>-229.4</td>
<td>7191.3</td>
<td>6408.7</td>
</tr>
<tr>
<td>65</td>
<td>-756.9</td>
<td>-1237.4</td>
<td>9657.2</td>
<td>8874.6</td>
</tr>
</tbody>
</table>

Table IV presents the performance comparison between Scheme 1 and Scheme 2 for different values of the regular retail price. Payoff 1, Payoff 2 represent total payoffs of DR aggregators; Profit 1, Profit 2 indicate the optimal profit values of the LSE; and DR1 and DR2 represent the total energy consumption of DR aggregators over the scheduling horizon for Scheme 1 and Scheme 2, respectively. We can see from Table IV that the minimum energy consumption level of all DR aggregators is 201.6 MWh. Table IV also shows that the total payoff of DR aggregators as well as the optimal profit of the LSE in Scheme 1 are significantly larger than those in Scheme 2. Therefore, we can conclude that Scheme 1 outperforms Scheme 2 in terms of DR aggregators’ payoffs and LSE’s profit.

Furthermore, we can observe that the total energy consumption of DR aggregators over the scheduling horizon in Scheme 1 is greater than the minimum energy consumption requirement (i.e., 60% of the total flexible loads or 201.6 MWh) for regular retail prices of 47 $/MWh, 50 $/MWh, and 55 $/MWh, and is equal to the minimum level for regular retail prices of 60 $/MWh and 65 $/MWh. Similar observation can be drawn for Scheme 2. Additionally, for the same value of regular retail price, DR1 is greater than DR2 since DR prices in Scheme 1 are always smaller or equal to the regular retail price while DR prices in Scheme 2 are equal to the regular retail price.

Fig. 5 shows the optimal hourly DR prices over the scheduling horizon for different values of \(c_d^R\) and \(p_{\text{grid}}\). We can observe that DR price is very low during time slots 1-8, quite low for some period during time slots 9-16, and very high during time slots 17-24. Intuitively, the LSE would set a low DR price during some time slots to encourage DR aggregators to consume more energy. In addition, it can set a high DR price (i.e., close or equal to the regular retail price) to discourage DR aggregators from consuming energy.

There are several reasons for the LSE to set low DR price. First, when the grid price is low, the LSE would be interested in buying more energy from the main grid to serve its customers at a DR price between the grid price and the regular
retail price. Second, the grid price can vary significantly over the scheduling horizon, which offers opportunities for the LSE to arbitrate between low and high price periods. Therefore, the LSE sets low DR prices at some time slots and high at some other time slots to encourage load shifting from DR aggregators in order to reduce the importing cost of energy from the main grid. Also, DR aggregators can reduce their bills by shifting their loads to low DR price hours. Finally, if renewable energy generation is high, the LSE faces the power limit at the PCC (i.e., $P_{\text{grid}}$); hence, it would sell as much energy as possible as to its customers at low DR prices rather than curtailing the renewable energy surplus.

During hours 1-8, the marginal utility (benefit) of consuming load for DR aggregators is relatively low (i.e., 0.8 times the base-case marginal utility in Table III), and the grid price is low. Therefore, the optimal DR price is low during this period to encourage DR aggregators to consume more energy and to shift load to this period. During hours 9-16, although the marginal utility is the same as the base-case marginal utility, the grid price is high; therefore, only for some first hours in this period, DR prices are lower than the regular retail prices. During hours 17-24, the grid prices include both high price hours and low price hours; however, the marginal utilities of consuming energy achieved by DR aggregators are high, which are equal to 1.2 times the base-case utilities. DR prices are high during this period even during low price hours. This is because the benefits of consuming energy for DR aggregators outweigh the energy costs. Furthermore, we can observe that when $P_{\text{grid}}$ is 20 MW, DR prices during hours 1-8 tend to be lower than DR prices when $P_{\text{grid}}$ is 40 MW. As will be illustrated in Figs. 6(a), 6(b), 7, and 8, involuntary load curtailment occurs for some hours in time slots 17-24 when $P_{\text{grid}}$ is 20 MW. This explains why the DR prices are lower during hours 1-8 so as to encourage DR aggregators to shift their load to this period.

Figs. 6(a) and 6(b) present the total hourly load of DR aggregators over the scheduling horizon with the regular retail prices of 60 $/MWh and 65 $/MWh, respectively. For Scheme 2\(^4\), the total DR load is low during hours 1-8, higher during hours 9-16, and highest during hours 17-24. This is because the marginal utility prices of DR aggregators are lowest during hours 1-8, and highest during hours 17-24. Furthermore, the DR price in Scheme 2 is equal to the regular retail price (i.e., fixed); hence, DR aggregators have no incentive to shift their loads. On the other hand, DR load in Scheme 1 is significantly higher than that in Scheme 2 during hours 1-8, and DR load in Scheme 1 is generally lower than DR load in Scheme 2 during time slots 9-24. This demonstrates the effectiveness of the proposed scheme in shifting load in favor of the LSE. Moreover, we can observe that more load shifting occurs when $P_{\text{grid}}$ is 20 MW than when $P_{\text{grid}}$ is 40 MW. This is to reduce load curtailment when $P_{\text{grid}}$ is 20 MW since renewable energy generation during time slots 19-24 is low. When the regular retail price is 60 $/MWh, utility due to energy consumption tends to outweigh the energy cost; therefore, even if the LSE sets lower DR price during hours 1-8, DR aggregators still consume a significant amount of energy during hours 17-24.

On the other hand, when the regular retail price is 65 $/MWh, DR aggregators have more incentive to shift their loads to low DR price hours.

\(^4\)Note that results for Scheme 2 are independent of the grid limit $P_{\text{grid}}$. 

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Fig. 5. DR price

Fig. 6. DR load

Fig. 7. Involuntary load curtailment
The results in Fig. 7 can be explained by the results in Figs. 6(a) and 6(b). Renewable energy generation is generally low during hours 18-24 and the total load of DR aggregators is very high during hours 17-24. For Scheme 1, more load shifting occurs when the regular retail price is 65 $/MWh than that when the regular retail price is 60 $/MWh. As a result, Scheme 1 outperforms Scheme 2 in terms of involuntary load curtailment. The hourly power trading between the LSE and main grid is shown in Fig. 8. As we can observe, in Scheme 2, the LSE has to sell electricity to the main grid during hours 1-8 when the grid prices are low, and it has to import a large amount of energy during high price hours to serve load. However, in Scheme 1, the LSE imports electricity during low price hours 1-8; hence, it can reduce the amount of imported electricity during high price hours.

Figs. 9(a), 9(b) illustrate the impact of the minimum energy consumption requirement of DR aggregators on the optimal solution. Parameter \(\text{minDR} \) is the ratio between the minimum energy consumption \(E_d\) and the maximum total load of each DR aggregator \(d\). As we can observe, the optimal profit of the LSE tends to increase as \(\text{minDR}\) increases. This is because DR aggregators are forced to consume more energy as \(\text{minDR}\) increases. Furthermore, the total payoff of DR aggregators tends to decrease as \(\text{minDR}\) increases. When \(\text{minDR}\) is equal to 100%, the loads of DR aggregators become inflexible since DR aggregators have to consume the maximum energy level. Also, when \(\text{minDR}\) is smaller than 100%, Scheme 1 outperforms Scheme 2 in terms of optimal LSE profit as well as total payoff of DR aggregators.

In Figs. 10(a), 10(b), 11, and 12, we show the impact of renewable energy generation on the optimal solution where
$P_{grid}$ is 20 MW. Parameter $RESSF$ (Renewable Energy Source Scaling Factor) is a scaling factor to scale the base-case renewable energy generation profile in Fig. 4(b). As we can observe in Figs. 10(a), 10(b), Scheme 1 outperforms Scheme 2 in reducing renewable energy curtailment. The results in Fig. 10(a) can be explained by those in Fig. 11. Compared to Scheme 2, DR aggregators consume a significantly larger amount of energy in Scheme 1 during hours with high renewable energy generation and lower during hours with low renewable energy generation. Furthermore, DR aggregators consume much more energy as $RESSF$ is 3 than when $RESSF$ is 2.5. This is because DR prices are very low as $RESSF$ is 3, and DR aggregators consume more energy to increase their utilities. Utility due to energy consumption outweighs the energy cost in this case. Hourly DR prices are shown in Fig. 12.

![Fig. 12. DR price](image-url)

Finally, in Fig. 14, we consider the case where $P_{grid}$ is equal to 20 MW and there is no renewable energy (i.e., $P_{RES,a}^t = 0$, $\forall t$). Hourly power trading between the LSE and the main grid ($P_{grid}^t$), total hourly DR load of DR aggregators, and total hourly involuntary load curtailment in Scheme 1 and Scheme 2 are presented in Fig. 14. Due to load shifting from DR aggregators in Scheme 1, we can observe that the power exchange with the main grid in Scheme 1 is higher than that in Scheme 2 during low grid price hours 1-8, and it tends to be lower than that in Scheme 2 during high grid price hours 9-17. Furthermore, involuntary load curtailment in Scheme 1 is significantly lower than that in Scheme 2.

![Fig. 13. Comparison between Scheme 1 (S1) and Scheme 2 (S2)](image-url)

C. Complexity of Proposed Approach

In this paper, the proposed pricing model is formulated as a bilevel program which can be recast as a single level MILP by using appropriate approximation methods. The global optimal solution of MILP can be obtained efficiently by using branch and cut algorithms embedded in available commercial solvers [34]. The optimization problem $(64)$ is solved by CPLEX 12.4 [39] under GAMS [42] on a laptop with 3.5 GHz Intel Core i7-3370 CPU and 8 GB RAM. The optimal solution is obtained with an optimality gap of 0.1%. The computation time with respect to number of DR aggregators is summarized in Table V. For fairness, the power limit at PCC ($P_{PCC}^{max}$) is set to be 500 MW to ensure that the LSE’s load can be served when the number of DR aggregators increases. Obviously, as the number of aggregators increases, the number of binary variables, and number of columns and rows in the reduced MILP obtained by GAMS/CPLEX increases, which results in the increase in the computation time. However, the computation time with 7 DR aggregators is only about 28s, which is acceptable.

The computation burden of the MILP, however, depends on several factors, especially the number of binary variables and constraints. The computation time of our model depends on many factors such as number of DGs, number of batteries, and the number of DG aggregators since these elements determine the number of binary variables and constraints. For example, operational constraints of conventional DGs such as ramping rate, minimum ON/OFF time duration increase the number of binaries and constraints, which consequently increase the computation burden. However, in this paper, we focus on a small-scale LSE that provides electricity to a community or a small network located at one single node or at a few nodes. The number of generators and the number of DR aggregators are, therefore, expected to be small. Hence, the underlying MILP can be solved efficiently using GAMS/CPLEX within reasonable computation time.

For a larger-scale application (e.g., a LSE serving a large distribution network with a significant number of generators and loads), it can be very challenging to solve the resulted large scale MILP. In this case, state-of-the-art decomposition techniques for a large scale MILP or the application of evolutionary algorithms such as particle swarm optimization (PSO) can be employed to reduce the computational time. However, these issues will be the subject of our future work.
### D. Constraints of Distribution Network, Batteries, and DGs

For the simulation, the network of LSE is modeled using a widely used IEEE 6-bus system as in [41]. For simplicity, the network flow limits at each line are set sufficiently large (i.e., 15 MW) to avoid congestion. Furthermore, we assume that the LSE owns three DGs and one battery. The data of these components are available in our previous work [2]. The minimum power outputs of DGs are \( P_{i}^{\min} = 0.5 \) MW, both ramping up and ramping down are set to 0.5 MW, minimum ON/OFF duration is 2 h. DR aggregators 1, 2, and 3 are located at bus 3, 4, and 5, respectively. DG 1 with generation capacity of 2 MW is located at bus 6. The 1 MWh battery (\( SOC_{\min}^{\text{max}} = 0.2 \), \( SOC_{\max}^{\text{min}} = 0.9 \), \( SOC_{0} = 0.5 \), \( P_{c} = P_{d} = 0.1 \) MW) and two DGs 2 and 3 each of which has generation capacity of 1 MW, are located at bus 2. Other system data is the same as in the base case. The inflexible load in Fig. 4(b) is allocated evenly to buses 3, 4, and 5. The network’s line susceptance data is taken from MatPower software [47]. The considered system topology is presented in Fig. 15.

![IEEE 6-bus system](image)

Fig. 15. IEEE 6-bus system

Fig. 16(a) presents the outputs of DGs, which increases significantly in the period between time slots 14 and 21, which is associated with high grid electricity price and peak load period. In particular, during on-peak time with high grid price the LSE tends to increase its energy generated from DGs to reduce the energy drawn from the grid and to compensate for the increase of customers’ demand and the deficit of renewable energy generation.

The difference of power dispatch schedules of DGs in the two cases with and without consideration of ramp rate and minimum ON/OFF time constraints are shown in Fig. 16(a) and Fig. 16(b), respectively. It is revealed that without these technical constraints, the LSE has more flexibilities to adjust its on-site DGs’ power generation. When ramp rate and minimum ON/OFF time constraints are more relaxed, the outputs of DGs exhibit larger variation. The LSE tends to turn on all units all the time (the minimum power of each unit is \( P_{i}^{\min} = 0.5 \) MW) to reduce the energy drawn from the grid. Consequently, the profit of LSE is slightly better, e.g., it is equal to 102168$ with ramp rate and minimum ON/OFF duration constraints and equal to 105188$ when these inter-temporal constraints are ignored. Batteries and DGs add additional flexibility to the operation of the LSE. The increasing LSE’s generation’s flexibility enables the LSE to exploit the advantages of time-varying grid electricity price, which is illustrated in Fig. 17. For example, it can buy less energy from the grid or sell more energy to the main grid during periods with high grid price to improve its profit.

The computation time of the optimization problem considering distribution network constraints (IEEE 6-bus system), batteries, and ramp rate and minimum ON/OFF time con-

![Power dispatch of DGs](image)

(a) With ramp rate and minimum ON/OFF duration constraints

![Power exchange with the main grid](image)

(b) Without ramp rate and minimum ON/OFF duration constraints

Fig. 16. Dispatch of DGs

Fig. 17. Power exchange with the main grid
straints of DGs is 18.627s, which is slightly higher than the computation time reported in Table V (with 3 DR aggregators). This is due to the additional constraints including the network flow constraints and operation constraints of batteries and DGs, which increase the computation burden of the problem.

Our design in this paper aims at developing an efficient DR pricing scheme which is suitable for a LSE deployed over a small area. Hence, the number of nodes in the LSE’s network is assumed to be small, and in many cases, all entities in the system can be located at just one node (e.g., a LSE supplies electricity to a small town or a village). In fact, the computation burden of the proposed LSE optimization model depends mainly on the number of DR aggregators, DGs, and batteries since these added elements introduce more binary variables. The proposed optimization can be extended to a stochastic bilevel programming problem, which can be tackled by using the scenario-based optimization approach. Due to the space constraint, study of the stochastic problem is reserved for our future works. In the stochastic case, the computation burden of the stochastic mathematical programs with equilibrium constraints (MPECs) depends highly on the number of scenarios. However, the complexity of the stochastic MPEC problem can be reduced by using a novel coordinate decent algorithm [10] to decompose the stochastic problem by scenario, or by using Bender decomposition as suggested in [43], or heuristic evolutionary algorithms [44].

VI. CONCLUSION AND FUTURE DIRECTIONS

In this paper, we have proposed a novel operation framework for a LSE, which serves both flexible and inflexible loads. The proposed pricing scheme can be readily implemented since it is compatible with the existing pricing structure in the retail market. Extensive numerical results have shown that the proposed scheme helps increase the profit of the LSE, increase payoff for DR aggregators, reduce involuntary load curtailment, and renewable energy curtailment.

There are several directions that the proposed optimization framework can be further extended.

- First, there are various uncertain system parameters in the considered model such as renewable energy generation and grid electricity price. Addressing this uncertainty issue by using popular optimization techniques such as robust optimization [8], stochastic optimization [11], and model predictive control [26] is an interesting research topic for further works. The MPC-based design, however, must consider the feasibility and stability of the closed loop system.

- Reactive power management is an important technical issue in the distribution network. Our future work will consider how the reactive power management can be integrated into the proposed optimization framework.

- The complexity of the proposed model increases as the number of DR aggregators increases. Additionally, the computational burden of the model increases significantly if we consider a scenario-based stochastic model to tackle the system uncertainties. The proposed model is computationally tractable when the number of DR aggregators and/or the number of scenarios is moderate, which is a reasonable assumption for the setting where the LSE serves a small area. It is interesting to study how one can reduce the computational time of the proposed model.

- There could be several LSEs operating in one (large) geographical area, and DR aggregators can choose the best LSE based on the offered DR prices. Therefore, LSEs need to determine an optimal DR pricing offer to attract more DR loads while maximizing their profits. The problem then becomes a multi-leader multi-follower game, which will be considered in the future.

REFERENCES


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