Joint Rate and Power Allocation for Cognitive Radios in Dynamic Spectrum Access Environment

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Abstract—We investigate the dynamic spectrum sharing problem among primary and secondary users in a cognitive radio network. We consider the scenario where primary users exhibit on-off behavior and secondary users are able to dynamically measure/estimate sum interference from primary users at their receiving ends. For such a scenario, we solve the problem of fair spectrum sharing among secondary users subject to their QoS constraints (in terms of minimum SNIR and transmission rate) and interference constraints for primary users. Since tracking channel gains instantaneously for dynamic spectrum allocation may be very difficult in practice, we consider the case where only mean channel gains averaged over short-term fading are available. Under such scenarios, we derive outage probabilities for secondary users and interference constraint violation probabilities for primary users. Based on the analysis, we develop a complete framework to perform joint admission control and rate/power allocation for secondary users such that both QoS and interference constraints are only violated within desired limits. Throughput performance of primary and secondary networks is investigated via extensive numerical analysis considering different levels of implementation complexity due to channel estimation.

Index Terms—Cognitive radio, spectrum overlay and spectrum underlay, rate and power allocation, quality of service (QoS), convex optimization.

I. INTRODUCTION

DYNAMIC spectrum sharing through cognitive radios can significantly enhance the spectrum utilization in a wireless network. There are two approaches for this dynamic spectrum sharing, namely, spectrum underlay and spectrum overlay approach (Chapter 3, [1]). The spectrum overlay increases the spectrum efficiency by granting secondary (i.e., unlicensed) users to opportunistically exploit unused frequency bands of primary (i.e., licensed) users when the frequency bands are sensed as being unused in temporal and spatial domains. In this scenario, it is not necessary to impose severe restrictions on the transmission power of the secondary users, since any interference from secondary user transmissions will not be harmful to the inactive primary users. But, this approach may not fully utilize the characteristics of co-channel interference in cellular environment and/or wideband signaling such as wideband code-division multiple access (WCDMA) and ultra-wideband (UWB) that can coexist with other systems because of low inter-user interference. In contrast, the spectrum underlay permits simultaneous sharing of all the frequency bands available among the primary and secondary users, while imposing severe restrictions on the transmission power of the secondary users, so as not to cause any harmful interference to the active primary users. In this scenario, we simply assume the worst-case primary user interference by treating the primary users all as being active. However, this approach prevents us from increasing the data rate of the secondary users when the primary users become inactive. Therefore, dynamic spectrum access of secondary users by adapting to on-off behavior of primary users could lead to further improvement in spectrum efficiency. For such a dynamic spectrum access environment, rate and power allocation algorithms for the cognitive radios need to be designed to achieve the desired network performance. This paper models and analyzes this spectrum sharing problem and proposes a solution to the dynamic rate and power allocation problem for secondary users.

Recently, there have been a flurry of works in the literature addressing different aspects of spectrum sensing, dynamic spectrum sharing, and spectrum pricing for cognitive radio networks [1]-[7]. The works related to resource allocation to secondary users under dynamic spectrum sharing are particularly relevant to our context. In a dynamic spectrum access environment with multiple primary and secondary users, the problem of determining the optimal number of secondary users relative to the number of primary users, which maximizes the sum throughput (primary and secondary), was addressed in [8]. The authors demonstrated by simulations the trade-off between sum throughput maximization and primary user interference minimization considering imperfect sensing and different levels of interference tolerance at the primary and secondary receivers. However, the interference dynamics and effects of channel fading due to the propagation environment were not considered. In [9], the implications of two fundamental concepts in dynamic spectrum access, namely, spectrum opportunity and interference constraints, were described in terms of communication activities of primary users in the neighborhood of secondary transmitters and receivers. Also,
for a dynamic spectrum access environment, the parameters required to specify the interference constraints and the parameters that affect the transmission power control of secondary users were identified. In [10], the problem of optimal power control for secondary users under interference constraints for primary users was formulated as a concave minimization problem. The authors proposed a branch and bound algorithm to obtain the solution. A transmit power control scheme for a secondary user (i.e., cognitive radio) was proposed in [11], which exploits the location information of the primary receiver obtained indirectly through spectrum sensing, to limit the interference caused to the primary receiver.

The problem of channel and power allocation for secondary users in a cellular cognitive radio network was addressed in [12]. In particular, the authors proposed a heuristic-based two-phase resource allocation scheme. In this scheme, channels and power are first allocated to the cognitive radio base stations to maximize their total coverage area while maintaining the interference constraints for primary users. Then each base station allocates the channels among the cognitive radios within its cell such that the total number of cognitive radios served is maximized. In [13], fair resource allocation solutions for the IEEE 802.22-like cognitive radio network was proposed. A novel multi-channel MAC protocol for opportunistic spectrum access (i.e., spectrum overlay) was developed in [14] where the spectrum sensing functionality was incorporated into the MAC protocol design. Throughput performance and collision probability with primary users due to spectrum sensing errors were derived and evaluated.

In [16], the problem of dynamic spectrum access with QoS guarantee (in terms of minimum required signal-to-interference ratio) for secondary users was studied under an interference temperature constraint for primary users. The problem was formulated as a convex geometric program where the globally optimal solution could be obtained under a feasible power allocation for the secondary users. Also, a centralized tree-pruning algorithm for removing secondary links was proposed for the cases where a feasible power allocation among all the secondary users is not possible. In [17], a distributed power and admission control algorithm that minimizes the total transmitted power (primary and secondary users) in a cognitive radio system was proposed. The objective of the proposed algorithm was to satisfy the target signal-to-interference-pus-noise ratio (SINR) requirements of all users. With a similar spirit, in [18], algorithms for joint rate and power allocation as well as admission control algorithms were proposed for cognitive radios in a spectrum underlay scenario which guarantee the minimum SINR and rate constraints for secondary users and the interference constraints for primary users. Maximum transmit power constraint and the fairness performance for the secondary users were considered.

In this paper, we present solutions for the spectrum sharing problem in a dynamic spectrum access environment. We consider the case where only mean channel gains from secondary users to primary receiving points averaged over short-term fading are available while either instantaneous or mean channel gains for the links among secondary users are available. Assuming that secondary users can dynamically track the sum interference from primary users at their receiving sides, we are interested in finding optimal resource allocation solutions subject to QoS constraints, minimum rate requirements for secondary users, and interference constraints for primary users. In particular, we propose a complete framework for joint admission control, rate/power allocation for optimal spectrum sharing in a cognitive radio network. The conference version of this paper was published in [15]. The major contributions of this paper can be summarized as follows:

- The notions of outage probability for QoS constraints for secondary users and violation probability for interference constraints for primary users are introduced. Analytical models are developed for these outage and violation probabilities considering the interference and fading dynamics in the system.
- Based on the aforementioned analysis, joint admission control, rate/power allocation method for cognitive radios is presented subject to QoS and minimum rate requirements as well as maximum transmit power and fairness constraints for secondary users. Also, inter-play among interference, fading dynamics and the resource allocation among cognitive radios is revealed.
- Numerical results show that the proposed framework can result in significant throughput enhancement in a dynamic spectrum access environment. Impacts of QoS and interference requirements as well as throughput performance under different levels of implementation complexity due to channel gain estimation are investigated through extensive numerical analysis.

The rest of the paper is organized as follows. Section II describes the system model and related assumptions. Section III presents the QoS, the interference constraints, and the problem formulation for resource allocation problem. The joint admission control and rate/power allocation method for cognitive radios is presented in Section IV. Section V presents the performance evaluation results. Conclusions are stated in Section VI.

II. SYSTEM MODEL AND SUM INTERFERENCE OF SECONDARY USERS

We consider a cellular wireless network where primary users communicate with the corresponding base stations through uplink transmission [19]. The secondary users (i.e., cognitive radios), which share radio spectrum with the primary users, communicate with each other in an ad-hoc mode. We will call a communication link between a pair of secondary users a secondary link in the sequel. We assume that primary users exhibit on-off behavior so the total traffic load contributed by all primary users varies with time depending on how many primary users are in the “on” state. By tracking the activity of the primary users through measuring the sum interference at the receiving sides of secondary links, better spectrum sharing between primary and secondary users can be achieved.

For the above dynamic spectrum access scenario, rate and power allocations for the cognitive radios are performed such that the following constraints are satisfied -

1) the QoS requirement for a secondary link in terms of signal-to-interference ratio (SIR) and minimum data rates;
2) the tolerable interference limit at the primary receiving points (i.e., base stations).
A secondary node would adjust its transmit power and rate so that the interference temperature limits at the primary receivers are not violated and its QoS requirements are satisfied. It is assumed that a central controller in the secondary users' network gathers information about the primary sum-interference, as well as the channel status of potential secondary links. Then, the controller performs the joint admission control, and rate/power allocation for secondary links, considering the QoS and the interference constraints, provided that the primary sum-interference can be measured properly at the secondary receiving nodes.

We now make some further assumptions about channel gain information. While it may be possible to estimate instantaneous channel gains among secondary users, it is more difficult to estimate instantaneous channel gains from secondary transmit users to primary receiving points. However, by exploiting pilot signals transmitted from primary BSs, secondary transmit users can estimate the mean channel gains from primary BSs to themselves. Due to the reciprocal characteristic of the wireless channel, these mean channel gains would be equal to the mean channel gains from secondary transmit users to primary BSs. For channel gains among secondary users, instantaneous or mean channel gains may be available depending on design sophistication of secondary mobile units. In any case, when only mean channel gains are available, we assume that these mean channel gains are averaged over short-term fading, and therefore, only the effects of long-term shadowing and path-loss are reflected in these gains. Moreover, channel gains corresponding to secondary user links are assumed to be reported to the controller in a slotted mode.

We consider an M-cell layout as in Fig. 1, where K primary users are uniformly distributed in the considered geographical area, communicating with their base stations centered in each cell, and there are N secondary communication links in the area. We assume that the signal format of the primary users is known a priori to the receiving nodes of secondary links. Based on this, the primary users' sum-interference is estimated and measured at the receiving node of secondary link \( i \) as

\[
N_i = \sum_{k=1}^{K} \varphi_k P_k^{(u)} g_{i,k}^{(u)}, \quad i = 1, 2, \ldots, N. \tag{1}
\]

Here, \( \varphi_k \in \{0, 1\} \) represents the \( k \)th primary user activity, \( g_{i,k}^{(u)} \) is the uplink channel gain from the \( k \)th primary user to the receiving node of secondary link \( i \), and \( P_k^{(u)} \) is the transmission power of the \( k \)th primary user in uplink direction. The instantaneous transmit power of primary user \( k \) associated with base station \( j \), \( P_k^{(u)} \), is given by

\[
P_k^{(u)} = \frac{P_i}{g_{j,k}^{(u)}} \tag{2}
\]

assuming an equal received power \( P_i \) at the primary receiving points (i.e., base stations) \( j \) from the corresponding primary users \( (j = 1, 2, \ldots, M) \) due to power control. Here, \( g_{j,k}^{(u)} \) is the instantaneous channel gain from the \( k \)th primary user to primary receiving point \( j \). We assume that the receiving node of secondary link \( i \) can estimate/measure the sum interference \( N_i \) and report it to the controller to perform resource allocation in each time slot.

Note that it would be more natural to let \( P_i \) vary according to the primary traffic. However, since there is no mechanism for the users to inform others about their “on/off” states, one particular user in “on” state does not know who among the other users are in the “on” state as well at any time to figure out the corresponding \( P_i \). This is a problem in dynamic adaptation of \( P_i \). If the primary receiving points (i.e., BSs) can estimate the fluctuations of the primary user traffic, then they would inform their mobiles of these changes so that the corresponding \( P_i \) could be set at each mobile in an optimal manner. Also, when the primary and the secondary systems co-exist, the above estimation may cause an extra overhead. An efficient method of adapting \( P_i \) could be obtained by measuring the overall interference caused by the two systems - this would be an interesting topic for further research.

III. CONSTRAINTS AND FORMULATION OF RESOURCE ALLOCATION PROBLEM

A. QoS Constraints for Secondary Users

As mentioned before we would like to perform resource allocation for secondary users subject to the QoS and interference constraints which will be defined below. For analytical purpose, we limit our framework to a CDMA-based wireless network where the primary and secondary users can transmit simultaneously in a common frequency band, but the framework developed here can easily be extended to other types of wireless networks with slight modifications.

The signal-to-interference ratio (SIR) at the receiving node of secondary link \( i \) can be formulated as

\[
\mu_i = \frac{B}{R_i} \sum_{j=1, j \neq i}^{N} g_{i,j}^{(s)} P_j + N_i
\]

where \( B \) is the system bandwidth, \( R_i \) and \( P_i \) are the transmission rate and power of secondary link \( i \), respectively, and \( g_{i,j}^{(s)} \) denotes the channel gain from the transmitting node of secondary link \( j \) to receiving node of secondary link \( i \).

Here, the processing gain \( B/R_i \) should be sufficiently large to suppress the inter-user interference caused by other users sharing the common channel. However, it can be assumed to be one for other multiple access technologies such as FDMA. Note that the background noise can be ignored in an interference-limited CDMA network. If the noise power is included, then \( N_i \) corresponds to the sum of noise power and sum-interference due to the primary users. The channel gain \( g_{i,j}^{(s)} \) can be decomposed into

\[
g_{i,j}^{(s)} = \bar{g}_{i,j}^{(s)} + a_{i,j}^{(s)}
\]

where \( \bar{g}_{i,j}^{(s)} \) is the local average of \( g_{i,j}^{(s)} \) and \( a_{i,j}^{(s)} \) represents short-term fading with mean value normalized to one.

Depending the design sophistication of secondary users, secondary users can estimate channel gains among secondary links instantaneously or in an average sense only, that is, by averaging over short-term fading. If instantaneous channel gains are available, the following SIR constraints

\[
\mu_i \geq \gamma_i, \quad i = 1, 2, \ldots, N \tag{4}
\]
must be satisfied by the resource allocation solutions where $\gamma_i$ is the required SIR corresponding to the desired value of bit error rate (BER). If only mean channel gains are available, the locally averaged SIR $\bar{\mu}_i$ at the receiving node of secondary link $i$ can be expressed as

$$\bar{\mu}_i = \frac{B}{R_i} \sum_{j=1}^{N_i} \frac{s_{i,j}}{\bar{g}_{i,j} P_j + N_i}.$$  \hfill (5)

In this situation, the average-sense QoS requirement considered here should conservatively be set as

$$\bar{\mu}_i \geq \alpha \gamma_i, \quad i = 1, 2, \ldots, N$$  \hfill (6)

for some $\alpha > 1$. In particular, the factor $\alpha$ needs to be determined a priori to constrain the outage probability of $\bar{\mu}_i < \gamma_i$ to a certain minimum level, denoted by $\delta(s)$, such that

$$\Pr[\mu_i < \gamma_i | \bar{\mu}_i \geq \alpha \gamma_i, N_i] \leq \delta(s), \quad i = 1, 2, \ldots, N. \quad (7)$$

Note that a proper value of $\alpha$ depends on the primary user activity factor and the fading channel statistics. Therefore, the value of $\alpha$ may need to be updated periodically.

**Proposition 1**: The outage probability for the average-sense QoS requirement given the measurement of $N_i$ is evaluated as

$$\Pr[\mu_i < \gamma_i | \bar{\mu}_i \geq \alpha \gamma_i, N_i] = 1 - \exp \left[ \frac{R_i \gamma_i N_i}{B \bar{g}_{i,i} P_i} \right] \times \left[ 1 - \sum_{j=1}^{N_i} \frac{\pi_j(s)}{1 + \frac{B \bar{g}_{i,j} P_j}{R_i \gamma_i \bar{g}_{i,i} P_i}} \right].$$  \hfill (8)

where the short-term fading gain $\sqrt{\bar{g}_{i,j}}$ is assumed to be Rayleigh distributed and the probability density function (pdf) of $\bar{g}_{i,j}$ is given by $f_{\bar{g}_{i,j}}(x) = e^{-x}$ (i.e., exponentially distributed), and

$$\pi_j(s) = \prod_{i=1}^{N_i} \frac{\bar{g}_{i,j} P_j}{\bar{g}_{i,i} P_j - \bar{g}_{i,j} P_j}.$$  

**Proof**: In the SIR formula in (3) let define $X_{i,j} = s_{i,j} P_j$, which is exponentially distributed with p.d.f. $f_{X_{i,j}}(x) = \frac{1}{\bar{g}_{i,j} P_j} e^{-x/\bar{g}_{i,j} P_j}$. Then, it can be shown [24] that $Y_i = \sum_{j=1}^{N_i} X_{i,j}$ has the p.d.f.

$$f_{Y_i}(y) = \sum_{j=1}^{N_i} \frac{\pi_j(s)}{\bar{g}_{i,j} P_j} e^{-y/\bar{g}_{i,j} P_j}.$$  \hfill (9)

Conditioned on the primary sum-interference $N_i$, the outage probability is evaluated as

$$\Pr[\mu_i < \gamma_i | \bar{\mu}_i \geq \alpha \gamma_i, N_i] = \int_0^{\infty} \left[ \int_{\max(0,B/\bar{g}_{i,j} P_j - N_i)}^{\infty} f_{Y_i}(y) \, dy \right] f_{X_{i,j}}(x) \, dx.$$  \hfill (10)

It can be further evaluated as

$$\Pr[\mu_i < \gamma_i | \bar{\mu}_i \geq \alpha \gamma_i, N_i] = \int_0^{R_i \gamma_i N_i/B} f_{X_{i,j}}(x) \, dx + \int_{R_i \gamma_i N_i/B}^{\infty} \left[ \frac{f_{Y_i}(y)}{B} \right] f_{X_{i,j}}(x) \, dx.$$  \hfill (11)

where

$$\int_0^{R_i \gamma_i N_i/B} f_{X_{i,j}}(x) \, dx = 1 - \exp \left[ -\frac{R_i \gamma_i N_i}{B \bar{g}_{i,i} P_i} \right].$$  \hfill (12)

$$\int_{R_i \gamma_i N_i/B}^{\infty} \left[ \frac{f_{Y_i}(y)}{B} \right] f_{X_{i,j}}(x) \, dx = \sum_{j=1}^{N_i} \frac{\pi_j(s)}{1 + \frac{B \bar{g}_{i,j} P_j}{R_i \gamma_i \bar{g}_{i,i} P_i}} \exp \left[ -\frac{R_i \gamma_i N_i}{B \bar{g}_{i,i} P_i} \right].$$  \hfill (13)

Therefore, the outage probability conditioned on $N_i$ is derived as

$$\Pr[\mu_i < \gamma_i | \bar{\mu}_i \geq \alpha \gamma_i, N_i] = 1 - \exp \left[ -\frac{R_i \gamma_i N_i}{B \bar{g}_{i,i} P_i} \right] \times \left[ 1 - \sum_{j=1}^{N_i} \frac{\pi_j(s)}{1 + \frac{B \bar{g}_{i,j} P_j}{R_i \gamma_i \bar{g}_{i,i} P_i}} \right].$$  \hfill (14)

**B. Interference Constraints for Primary Users**

Let $\bar{g}_{j,i}^{(p)}$ denote the channel gain from the transmitting node of secondary link $i$ to primary receiving point $j$ and $\bar{g}_{j,i}^{(p)}$ be its mean value averaged over short-term fading. Also, let $I_j$ be the maximum interference limit tolerable at primary receiving point $j$. Then, the interference constraints can be written as

$$\eta_j = \sum_{i=1}^{N} \bar{g}_{j,i}^{(p)} P_i \leq I_j, \quad j = 1, 2, \ldots, M. \quad (15)$$

As mentioned before, the instantaneous channel gains $\bar{g}_{j,i}^{(p)}$ may be difficult to estimate in practice. We assume that only the mean channel gain $\bar{g}_{j,i}^{(p)}$ can be estimated by processing the pilot signal from the primary receiving point $j$, where the long-term fading appears identical for both uplink and downlink due to the reciprocity of the channel. With only mean channel gains, interference constraints can be set in an average sense as follows:

$$\bar{\eta}_j = \sum_{i=1}^{N} \bar{g}_{j,i}^{(p)} P_i \leq \beta I_j, \quad j = 1, 2, \ldots, M \quad (16)$$

for some $\beta < 1$. Since the instantaneous interference level $\eta_j = \sum_{i=1}^{N} \bar{g}_{j,i}^{(p)} P_i$ may exceed the tolerable limit $I_j$, and therefore, violate the absolute interference constraint, i.e., $\eta_j \leq I_j$, we define a constraint on the violation probability as follows:

$$\Pr[\eta_j > I_j | \bar{\eta}_j \leq \beta I_j] \leq \delta(I), \quad j = 1, 2, \ldots, M \quad (17)$$

1For a particular modulation and coding assumed there exists an explicit relationship between the BER and target SIR.
where \( \delta^{(j)} \) denotes the maximum interference violation probability allowed for primary receiving points.

**Proposition 2:** The violation probability for the average-sense interference constraint is evaluated as

\[
\Pr[\eta_j > I_j | \eta_j \leq \beta I_j] = \frac{1}{\beta I_j} \sum_{i=1}^{N} \frac{\bar{g}_{j,i}^{(p)} P_i}{\bar{g}_{j,i}^{(p)} P_i - g_{j,i}^{(p)} P_i} 
\]

subject to \( \eta_j \leq \beta I_j \) where the short-term fading \( a_{j,i}^{(p)} = g_{j,i}^{(p)} / \bar{g}_{j,i}^{(p)} \) is exponentially distributed with pdf \( f_{\eta_j}^{(p)}(x) = e^{-x}, \) and

\[
\bar{g}_{j,i}^{(p)} = \prod_{i=1}^{N} \frac{g_{j,i}^{(p)} P_i}{g_{j,i}^{(p)} P_i - g_{j,i}^{(p)} P_i}.
\]

**Proof:** First, the tolerable interference limit \( \eta_j \) can be expressed as

\[
\eta_j = \sum_{i=1}^{N} g_{j,i}^{(p)} P_i = \sum_{i=1}^{N} a_{j,i}^{(p)} \frac{g_{j,i}^{(p)} P_i}{g_{j,i}^{(p)} P_i} P_i.
\]

Similar to the random variable \( Y_i \) in (9), \( \eta_j \) has the p.d.f.

\[
f_{\eta_j}^{(p)}(y) = \sum_{i=1}^{N} \frac{\bar{g}_{j,i}^{(p)} P_i}{g_{j,i}^{(p)} P_i} e^{-y/g_{j,i}^{(p)} P_i}.
\]

Therefore, the violation probability \( \Pr[\eta_j > I_j | \eta_j \leq \beta I_j] \) can be derived as given in (18).

In summary, to ensure that the average-sense QoS and interference constraints given by

\[
\bar{\mu}_i \geq \alpha \gamma_i \quad \text{and} \quad \bar{\eta}_j \leq \beta I_j
\]

are effective, the two parameters \( \alpha > 1 \) and \( \beta < 1 \) should be determined a priori to meet the constraints on outage probability and violation probability derived earlier.

**C. Problem Formulation**

We are interested in finding fair resource allocation solutions for secondary users subject to constraints described in the previous subsections. Specifically, we will solve the following optimization problem.

\[
\max \sum_{i=1}^{N} \ln(R_i)
\]

subject to

\[
\begin{align*}
R^\text{min} \leq R_i \leq R^\text{max}, & \quad i = 1, \ldots, N \\
0 \leq P_i \leq P^\text{max}, & \quad i = 1, \ldots, N
\end{align*}
\]

where the objective function \( \sum_{i=1}^{N} \ln(R_i) \) provides the well-known proportional fair solutions [21] of data rates for different secondary links. Note that either QoS constraints stated in (4) or (6) are used depending on whether secondary users can estimate instantaneous and mean channel gains to/from one another. Moreover, due to the minimum rate constraints for each secondary link, the problem may not be feasible. Therefore, a joint admission control and rate/power allocation should be performed to find a feasible solution of this problem.

**IV. SOLUTIONS OF RESOURCE ALLOCATION PROBLEM**

In this section, we present a solution approach for the resource allocation problem described in the previous section. We assume that in each time slot the primary network provides its tolerable interference limit \( I_j \) to the secondary central controller based on its current traffic load and other system/design parameters. Based on the tolerable interference limit \( I_j \) and the sum interference from primary users \( N_i \), the secondary network controller will find optimal solution for the problem stated in (21)-(23). In essence, the secondary network can dynamically share the spectrum with primary users by adapting to current traffic load in the network.

**A. Design of Primary Network**

First, to observe the interaction between the interference limit \( I_j \) and the achieved SIR at the primary receiving point \( j \), we express the corresponding SIR as

\[
\mu_j^{(p)} = \frac{B}{R} \frac{P_r}{(1 + f) \sum_{k=2}^{K_i} \varphi_k P_r + I_j}, \quad j = 1, 2, \ldots, M
\]

where \( R \) is the data rate for a primary user (e.g., for voice service), \( f \) denotes the frequency reuse factor, and \( K_i \) is the number of primary users served by the receiving point \( j \) with \( \sum_{j=1}^{M} K_j = K \).

In fact, the impacts of secondary links are captured in \( I_j \) which is the interference they create for primary users. Specifically, the interference from primary users should be smaller than \( I_j \) with high probability. In essence, the interference term \( I_j \) plays the role of Gaussian noise in the traditional cellular network. Therefore, it is natural to use frequency reuse factor to represent the impact of primary interference of other cells to one particular cell as in the traditional cellular network.

If the achieved SIR is controlled in an average sense for a target SIR of \( \gamma_j^{(p)} \), that is,

\[
\mu_j^{(p)} = \frac{B}{R} \frac{P_r}{(1 + f)(K_j - 1)pP_r + I_j} \geq \kappa \gamma_j^{(p)}
\]

where \( p \) is the activity factor (i.e., the probability that a primary user is in “on” state) and \( \kappa > 1 \) is a design parameter similar to \( \alpha \) in (6). Then the tolerable interference limit is obtained as

\[
I_j = \frac{B}{R} \frac{P_r}{\kappa \gamma_j^{(p)} - (1 + f)(K_j - 1)pP_r}.
\]

Here, we need to find \( I_j \) and \( \kappa \) such that the probability of violation of SIR requirements for the primary users remains below an outage probability threshold. Specifically, given \( I_j \), the value of \( \kappa \) can be found by evaluating the outage probability as

\[
P_{\text{out}} = \Pr[\mu_j^{(p)} < \gamma_j^{(p)} | I_j(\kappa)], \quad j = 1, 2, \ldots, M
\]

where \( \gamma_j^{(p)} \) is the design parameter as defined in Proposition 3. Specifically, we will search the conservative factor \( \kappa \) such that

\[
P_{\text{out}} \leq \delta^{(p)}
\]
where the outage probability is given in (27).

**Proposition 3:** The probability for the achieved SIR at the primary receiving point \( j \) being greater than or equal to the target SIR \( \gamma^{(p)} \) can be evaluated as a function of the parameter \( \kappa \) as follows:

\[
Pr\left[\mu_j^{(p)} \geq \gamma^{(p)}, \ n \text{ out of } K_j \text{ is active}\right] = \left(\frac{K_j}{n}\right) p^n (1-p)^{K_j-n} u\left[\Delta(\kappa) - n + 1\right]
\]

where \( u[n] = 1 \) for \( n \geq 0 \) and zero otherwise, and

\[
\Delta(\kappa) = \left\lfloor \frac{B}{R} \left(1 + \frac{1}{\gamma^{(p)}}\right) \left(1 - \frac{1}{\kappa}\right) + p(K_j - 1)\right\rfloor
\]

where \( \lfloor x \rfloor \) denotes the integer part of \( x \).

**Proof:** Combining (24) and (26) with \( \mu_j^{(p)} \geq \gamma^{(p)} \) yields

\[
Pr\left[\mu_j^{(p)} \geq \gamma^{(p)}, \ n \text{ out of } K_j \text{ is active}\right] = Pr\left[\sum_{k=2}^{K_j} \varphi_k = n - 1 \leq \Delta(\kappa) \right] \ n \text{ out of } K_j \text{ is active}
\]

\[
= \frac{K_j}{n} \left[\Delta(\kappa) - n + 1\right].
\]

Therefore, the joint probability as given in (29) is evaluated as

\[
Pr\left[\mu_j^{(p)} \geq \gamma^{(p)}, \ n \text{ out of } K_j \text{ is active}\right] = \frac{K_j}{n} p^n (1-p)^{K_j-n}. \quad (30)
\]

**B. Joint Admission Control, Rate/Power Allocation**

Given the value of \( I_j \) which is calculated in (26), we can design both primary and secondary networks as follows:

**Algorithm 1:** Joint design of primary and secondary networks

1. Assuming that the values of \( R \) and \( p \) for the primary users are given, using (27), (28), the primary network can calculate the values of \( I_j \) and \( \kappa \) such that the SIR requirements of primary users are violated with a probability smaller than the outage probability threshold \( \delta^{(p)} \).
2. Given \( I_j \) calculated in step 1, the secondary network controller finds the solution of the joint admission control and rate/power allocation problem defined in (21)-(23).

Note that we need to recompute the joint admission control and rate/power allocation solution when the channel gains (i.e., \( \bar{g}_{j,i}^{(s)} \) or \( \bar{g}_{j,i}^{(s)} \)) and the number of primary, secondary users changes. If the average channel gains \( \bar{g}_{j,i}^{(s)} \) are used, the computation is repeated less frequently. Also in step 2 of the above algorithm, due to the minimum rate requirements for secondary links, the rate/power allocation problem may not be feasible. Hence, admission control should be jointly performed with rate/power allocation to push the network into an admissible region. To decompose the admission control from the rate/power allocation problem, we remove the minimum rate requirements and solve the rate/power allocation problem first (i.e., constraints in (22) become \( R_i \leq R_{\text{max}} \) without the lower bound).

To obtain rate/power allocation solutions, we need to determine conservative factors \( \alpha \) and \( \beta \) such that SIR and interference constraints are only violated with desired probabilities (i.e., these values are specified by \( \delta^{(s)} \), \( \delta^{(f)} \)). Unfortunately, violation probabilities of SIR and interference constraints presented in Propositions 1 and 2 depend on the rate and power vectors which are the solutions of the resource allocation problem. Due to the coupling of the design parameters, we propose the following iterative algorithm to find the conservative factors and resource allocation solutions.

**Algorithm 2:** Joint rate and power allocation with desired constraint violation probabilities

1. Initialize \( \alpha = 1, \beta = 1 \)
2. Solve the joint rate and power allocation problem with current values of \( \alpha \) and \( \beta \) as follows:

\[
\max_{\{P,R_i\}} \sum_{i=1}^{N} \ln(R_i) \quad (31)
\]

subject to

\[
\sum_{i=1}^{N} \bar{g}_{j,i}^{(p)} P_i \leq \beta I_j, \quad j = 1, 2, \ldots, M \quad (32)
\]

\[
\mu_i \geq \gamma_i \quad \text{or} \quad \bar{\mu}_i \geq \alpha \gamma_i, \quad i = 1, 2, \ldots, N \quad (33)
\]

\[
R_i \leq R_{\text{max}}, \quad i = 1, \ldots, N \quad (34)
\]

\[
0 \leq P_i \leq P_{\text{max}}, \quad i = 1, \ldots, N \quad (35)
\]

3. Calculate the violation probabilities for SIR and interference constraints using propositions 1 and 2 and check whether they are smaller than the desired values in (7) and (17). If yes, finish; otherwise go to step 4.
4. Adjust the conservative factors as follows. If only mean channel gains are available and one or more of the constraints in (7) are violated, perform the following update

\[
\alpha = \alpha + \Delta \alpha \quad (36)
\]

If one or more of the constraints in (17) are violated, perform the following update

\[
\beta = \beta - \Delta \beta \quad (37)
\]

where \( \Delta \alpha \) and \( \Delta \beta \) are small adjustment values.
5. Return to step 2.

Note that in (33), if instantaneous channel gains \( \bar{g}_{j,i}^{(s)} \) can be estimated by secondary users, we use constraints \( \mu_i \geq \gamma_i \); otherwise, if only mean channel gains \( \bar{g}_{j,i}^{(s)} \) are available, we use constraints \( \bar{\mu}_i \geq \alpha \gamma_i \). For the case where instantaneous channel gains \( \bar{g}_{j,i}^{(s)} \) can be estimated, we only need to search for conservative factor \( \beta \) (i.e., \( \alpha = 1 \)). Otherwise, if only mean channel gains \( \bar{g}_{j,i}^{(s)} \) are available, we have to search for both conservative factors \( \alpha \) and \( \beta \). The optimization problem stated (31)-(35) can be converted into the following problem which is a geometric convex program [25]. Hence, its optimal solution...
can be found.

\[
\begin{aligned}
\text{minimize} & \quad (1/N) \prod_{i=1}^{N} R_i \\
\text{subject to} & \\
\sum_{i=1}^{N} \frac{\alpha_{g_{j,i}^{(s)}}}{B_{g_{j,i}^{(s)}}} R_i & \leq 1, \quad j = 1, 2, \cdots, M \quad (38) \\
\frac{\alpha_{g_{j,i}^{(s)}}}{B_{g_{j,i}^{(s)}}} R_i P_i & \leq 1, \quad j = 1, 2, \cdots, M \quad (39) \\
(R_{i}^{\text{max}})^{-1} R_i & \leq 1, \quad (R_{i}^{\text{min}})^{-1} P_i \leq 1, \quad i = 1, 2, \cdots, N \quad (40)
\end{aligned}
\]

Note that the transformed problem in (38) corresponds to the case where only mean channel gains \( g_{j,i}^{(s)} \) available. For the case with instantaneous channel gains \( g_{j,i}^{(s)} \), a similar convex program can be obtained. Now, we consider the admission control problem to ensure minimum rate requirements for secondary links. We propose the following two algorithms to perform joint admission control, rate/power allocation.

**Algorithm 3: One-step removal algorithm**

1. Solve the rate/power allocation problem without minimum rate requirements using Algorithm 2.
2. Perform admission control using rate/power allocation solution in step 1 as follows. For each secondary link \( i \), compare optimal rate \( R_i^{\text{opt}} \) with minimum rate \( R_i^{\text{min}} \). Remove all secondary links with \( R_i^{\text{opt}} < R_i^{\text{min}} \).
3. Solve the rate/power allocation problem again for the remaining set of secondary links (i.e., secondary links for which the minimum rate requirements are satisfied) using Algorithm 2.

This algorithm is fast because we remove “violated links” in only one step. Therefore, we need to solve the rate/power allocation problem using Algorithm 2 at most two times. Algorithm 3 is fast but it may be too “greedy”. Another possible way to perform admission control is to remove secondary links one by one until we obtain a feasible set of secondary links as follows.

**Algorithm 4: One-by-one removal algorithm**

1. Solve the rate/power allocation problem without minimum rate requirements using Algorithm 2.
2. Remove at most one worst secondary link using rate/power allocation solution in step 1 as follows. If the optimal solution in step 1 is such that all secondary links achieve their minimum rates, finish. Otherwise, remove one link with the smallest rate (i.e., remove link \( i^* = \arg\min_i \{ R_i^* \} \)).
3. Solve the rate/power allocation problem again for the remaining set of secondary links and go to step 2.

It is evident that in the worst case, we have to perform rate/power allocation using Algorithm 2 \( N \) times. We will compare throughput and admission performance of these two algorithms in Section V.

### C. Throughput Performance

The sum of the transmission rates of the primary users \( R^{(p)} \) can be calculated as

\[
R^{(p)}(\kappa) = R \sum_{j=1}^{M} \sum_{n=1}^{K_j} n \Pr[\mu_j^{(p)} \geq \gamma^{(p)}, n \text{ out of } K_j \text{ is active}] 
\]

Therefore, the total transmission rate achieved by both primary and secondary users can be written as

\[
R^{(n)}(\kappa, \{ R_i^* \}) = R \sum_{j=1}^{M} \sum_{n=1}^{K_j} n \left( \frac{K_j}{n} \right) p^n (1-p)^{K_j-n} u \left[ \Delta(\kappa) - n + 1 \right] + \sum_{i=1}^{N} R_i^* \quad (42)
\]

where \( R_i^* \) is the solution of the joint admission control and rate/power allocation problem in Section V.B. Note that traffic load of the primary network will change the tolerable interference limit \( I_j \) in (26) which in turn affects throughput of the secondary network.

### V. PERFORMANCE EVALUATION

In this section, we present illustrative numerical results for the proposed resource allocation model. We consider a 3-cell layout scenario where in each cell \( K/M \) primary users are randomly located as in Fig. 1. In each time slot, locations of \( N/M \) pairs of secondary users are generated randomly in each cell. Specifically, locations of transmitting secondary users are generated randomly in each cell and their corresponding receivers are generated randomly within a distance of \( R/2 \) from the transmitters where \( R \) is the radius of each cell. The simulation results are obtained using channel/design parameters which are summarized in Table I. All performance measures are obtained by averaging over 100 simulation runs. Except for results in Figs. 7, 8, the voice activity \( p = 3/8 \) is used to obtain results in all other figures. The adjustment values for conservative factors in Algorithm 2 are chosen to be \( \Delta\alpha = 2 \), \( \Delta\beta = 0.05 \).
As presented in Section IV, we can choose the conservative factor $\kappa$ such that the outage probability in (28) is satisfied. In Fig. 2, we show the outage probability versus the conservative factor $\kappa$ for different number of primary users. The stepwise change of the outage probability in Fig. 2 can be interpreted as follows. As equation (24) shows, SINR of a primary user depends on the total interference from other primary users. Also, equation (26) says that $I_j$ decreases with increasing $\kappa$. The outage occurs for one primary user when total interference from other primary users is large enough. Since the number of primary users is an integer, the total aggregate interference has an inherent stepwise variation depending on how many primary users are in the “on” state. This explains the stepwise changes of the outage probability in Fig. 2. Note that, the behavior of the primary user interference patterns is determined by the Binomial distribution of the active primary users. In the limit, when the number of the primary users becomes very large, it approaches Poisson distribution which will create a more continuous-valued primary user interference pattern (resulting in continuous changes in the outage probability).

As can be seen, the desired outage probability can be achieved by choosing a value of $\kappa$ which is large enough. We can easily search for such a minimum value of $\kappa$. Given the value of $\kappa$, we can calculate the interference limit $I_j$ from (26) which will be used for the interference constraints (16). Here, the value of received power for primary users $P_r$ is calculated as follows:

$$P_r = 10^{16/10} RN_0$$

where $N_0$ is one-sided spectral density of Gaussian noise and this received power value $P_r$ achieves the signal-to-noise ratio (SNR) of 16 dB at the BS. With required SIR for primary users $\gamma^{(p)} = 6$ dB, we can calculate the interference limit $I_j$ assuming that the Gaussian noise is negligible.

We investigate the throughput performance of the joint admission control, rate/power allocation algorithms presented in Section V.B. Specifically, we compare the throughput performance of two admission control algorithms, namely, one-step removal and one-by-one removal algorithms. We will investigate throughput performance under two cases where either instantaneous channel gains $g^{(s)}_{j,i}$ or mean channel gains $\bar{g}^{(s)}_{j,i}$ among secondary users are available (denoted as instantaneous gains and average gains in all figures). Recall that if instantaneous channel gains $g^{(s)}_{j,i}$ are available, instantaneous SIR constraints for secondary links in (4) will be satisfied, and it is only required to search for the conservative factor $\beta$.

In Fig. 3, we show the throughput performance of the secondary network versus minimum processing gain under two admission control algorithms. As expected, throughput of the secondary network decreases when minimum processing gain increases. However, it is quite surprising that the one-by-one removal algorithm only achieves a little bit higher throughput than the one-step removal algorithm. Recall that one-step removal algorithm needs to run Algorithm 2 at most two times while the one-by-one removal algorithm may incur much higher computational
complexity. Also, the secondary throughput with instantaneous channel gains \( g_{j,i}^{(s)} \) among secondary users is significantly larger than that with mean channel gains \( \bar{g}_{j,i} \). Note, however, that in order to achieve high throughput, channel gains \( g_{j,i}^{(s)} \) should be estimated as fast as fading rate which is quite challenging in practice.

In Fig. 4, we plot the number of secondary links admitted for versus the number of requesting secondary links while we show throughput of the secondary network versus the number of secondary links in Fig. 5. Again, it is observed that both admission control algorithms, namely, Algorithms 3 and Algorithm 4, achieve similar performance in terms of throughput and number of secondary links admitted. Moreover, for the case with instantaneous channel gains, both performance measures continue to increase while these measures saturate at 12 requesting secondary links when only mean channel gains are available. Also, with 6 requesting secondary links, throughput of the secondary network for the case of instantaneous channel gains is less than two times that with mean channel gains. However, the performance gap in terms of throughput is more than two times with 15 requesting secondary links.

We show throughput of the secondary network versus the number of primary users in Fig. 6 for different values of \( \delta_s \) and \( \delta_I \). As expected, throughput performance decreases as the QoS and the interference constraint violation probabilities become more stringent. Also, throughput of the secondary network decreases with increasing number of primary users. This is because traffic load of the primary network increases with the number of primary users which in turn reduces the interference limit \( I_j \). Hence, throughput of the secondary network decreases. This figure shows that the secondary network can dynamically adapt to traffic load in the network and exploit the remaining unused capacity of the spectrum band.

We plot throughput of the secondary network versus the voice activity factor \( p \) in Fig. 7 and total throughput of both primary and secondary networks versus \( p \) in Fig. 8 under our proposed approach and worst case design where all primary users are treated as being always active. These figures show
that our adaptive approach, in which the dynamics of $N_i$ is tracked, achieves much higher throughput compared to the worst case design. Moreover, while throughput of the secondary network decreases, the total throughput of both networks tends to increase when the voice activity of primary users increases. This shows the capability of cognitive radios to squeeze more throughput from the underlying spectrum which would ultimately improve the spectrum utilization.

To investigate the fairness property of the resource allocation solutions for secondary links, we plot fairness index versus the minimum processing gains in Fig. 9 for both cases where instantaneous or mean channel gains among secondary users are available. The fairness index is calculated as $F_i = \left( \frac{\sum_{i=1}^{N} R_i}{N \sum_{i=1}^{N} R_i^2} \right)^2 [22]$. In particular, the fairness index becomes closer to one when resource allocation becomes fairer. As is evident, resource allocation solutions are a bit more fair for the case with mean channel gains. In general, resource allocation solutions are quite fair for both the cases and the fairness improves as the minimum processing gain becomes larger.

VI. Conclusion

We have proposed a framework for dynamic spectrum sharing between primary and secondary networks. Joint admission control and rate/power allocation schemes have been developed where the interference limits at primary receiving points are adapted depending on traffic load of the primary network. Since instantaneous channel gains among secondary links and those from transmitting secondary nodes to primary receiving points may be not be easy to estimate, we have derived outage probability for SIR constraints and violation probability for interference constraints considering fading dynamics of the wireless channel. Then, we have proposed a solution approach for the resource allocation problem. Numerical results have shown the efficacy of the proposed framework and revealed several interesting aspects of the resource allocation solutions. Several extensions of the proposed resource allocation framework are worth pursuing. The energy constraints in the secondary nodes (e.g., cognitive radio based sensor nodes) or the cost for transmission power can be considered in the joint rate and power allocation model. The resource allocation model can be extended for an ad hoc network formed by the primary users coexisting with the secondary users.

REFERENCES


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